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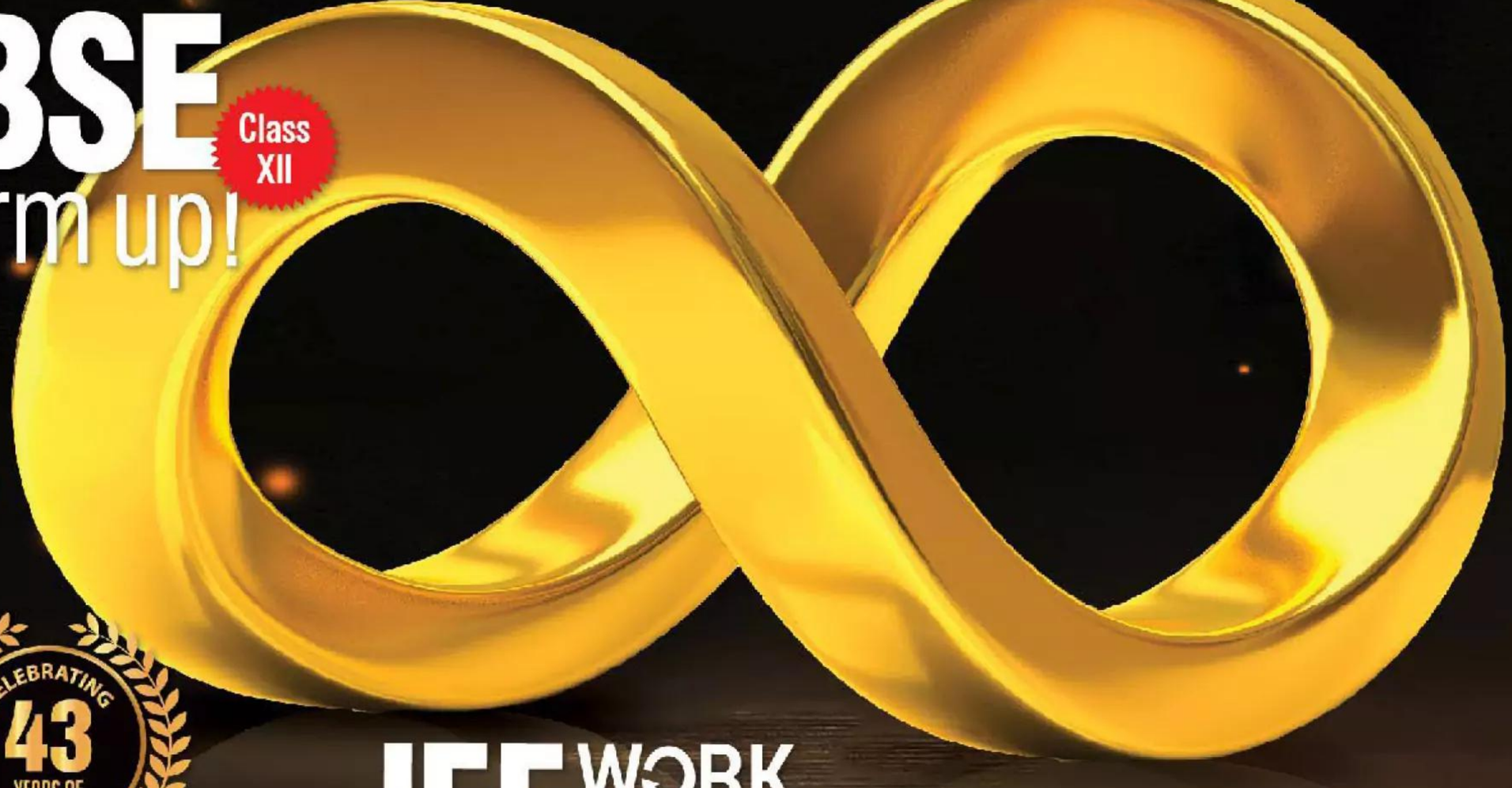

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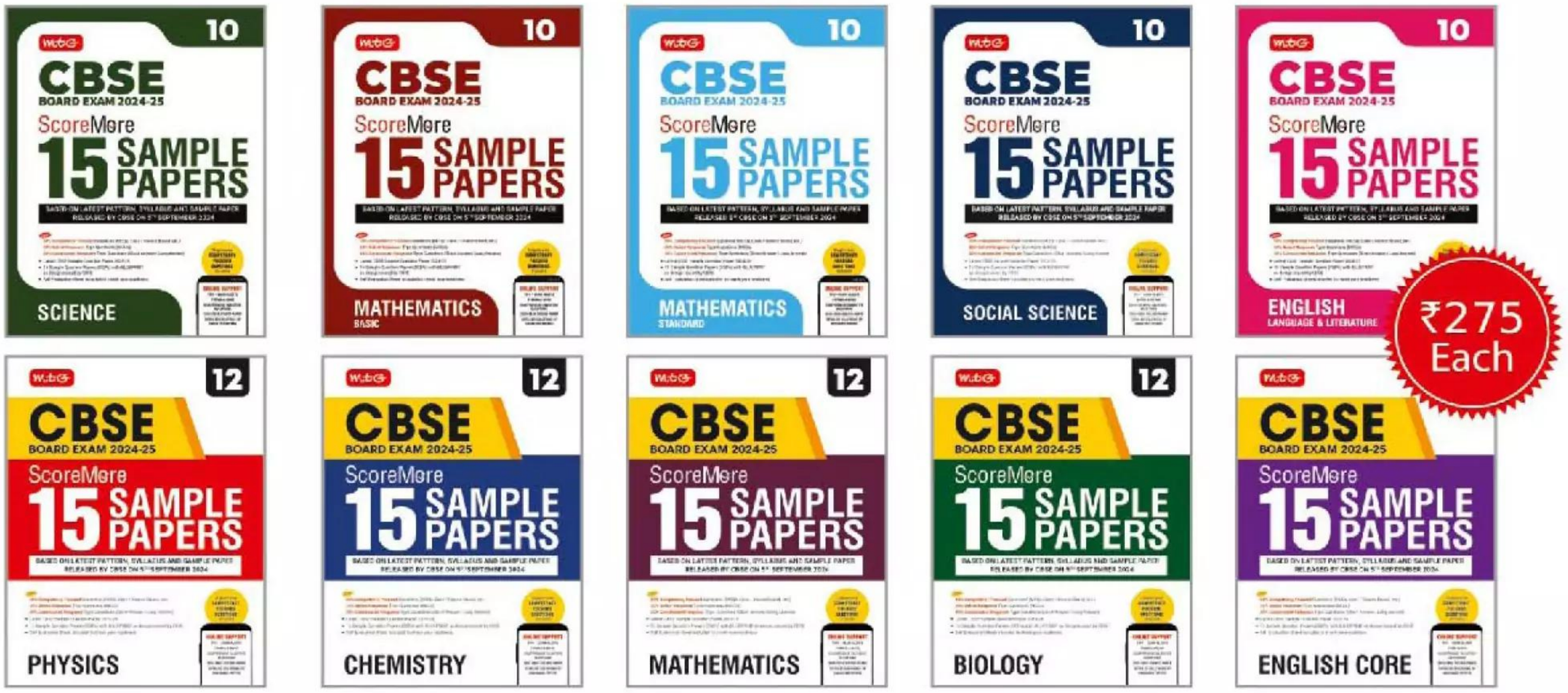

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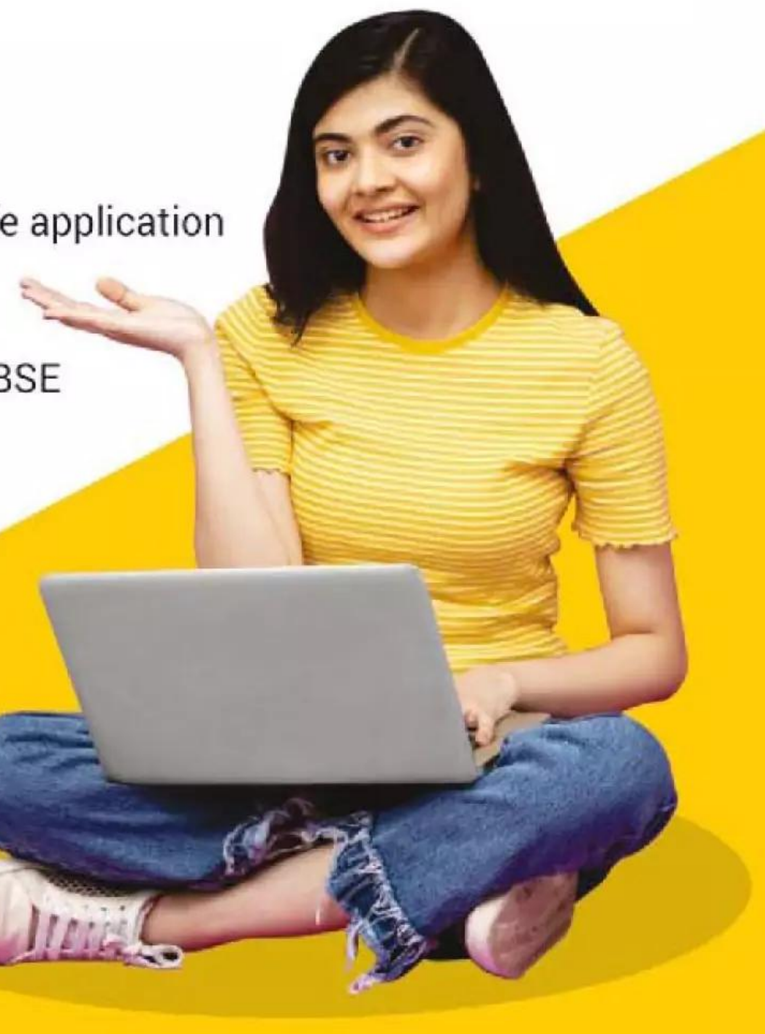

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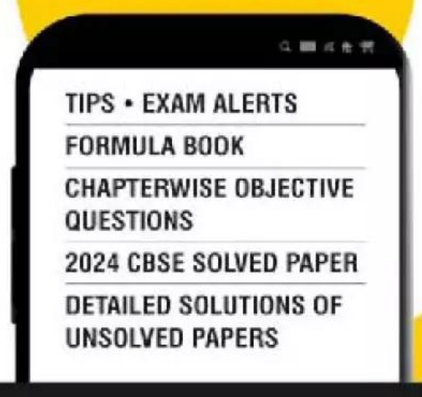

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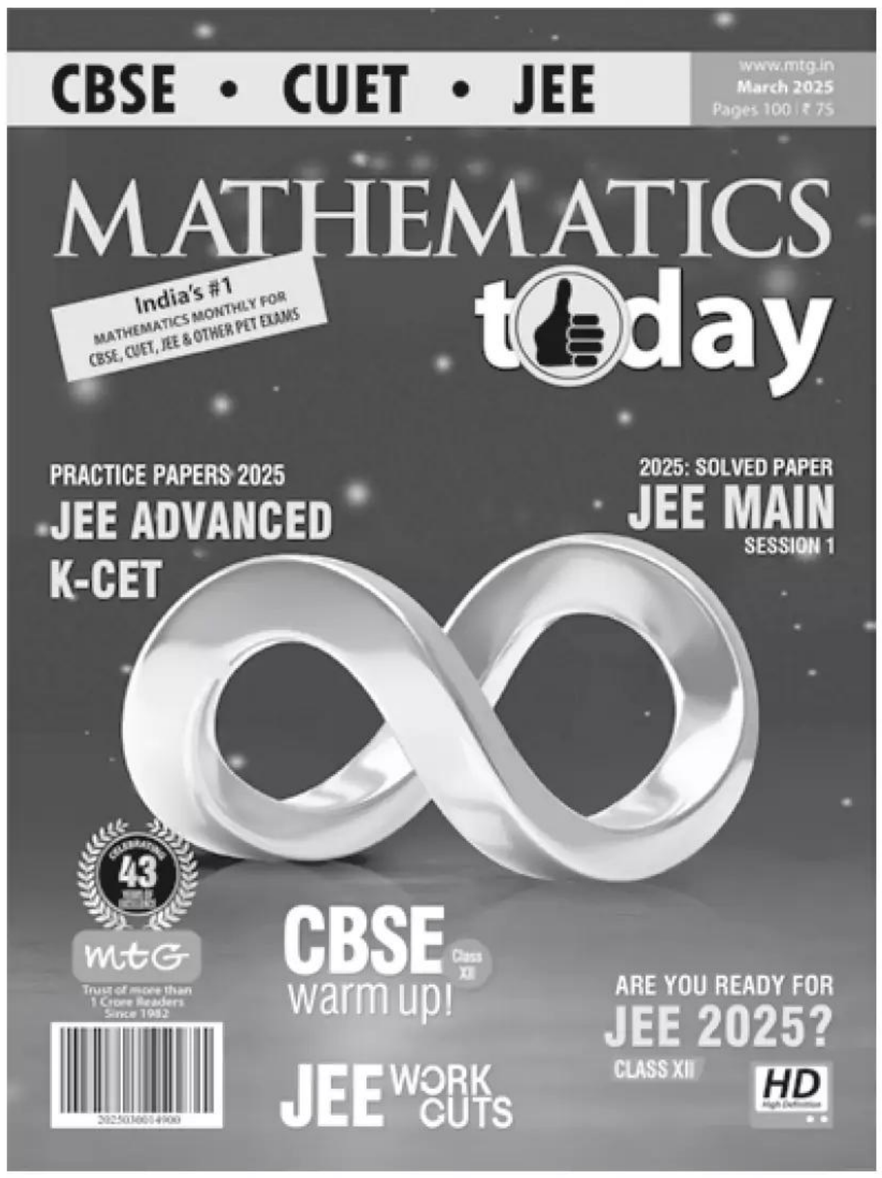

MATHEMATICS today

VOLUME 43 No. 3 March 2025

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Plot 99, Sector 44 Institutional area, Gurugram -122 003 (HR).
Tel : 0124-6601200 e-mail : info@mtg.in website : www.mtg.in

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Printed and Published by Mahabir Singh on behalf of MTG Learning Media Pvt. Ltd. Printed at HT Media Ltd., B-2, Sector-63, Noida, UP-201307 and published at 406, Taj Apartment, Ring Road, Near Safdarjung Hospital, New Delhi - 110029.
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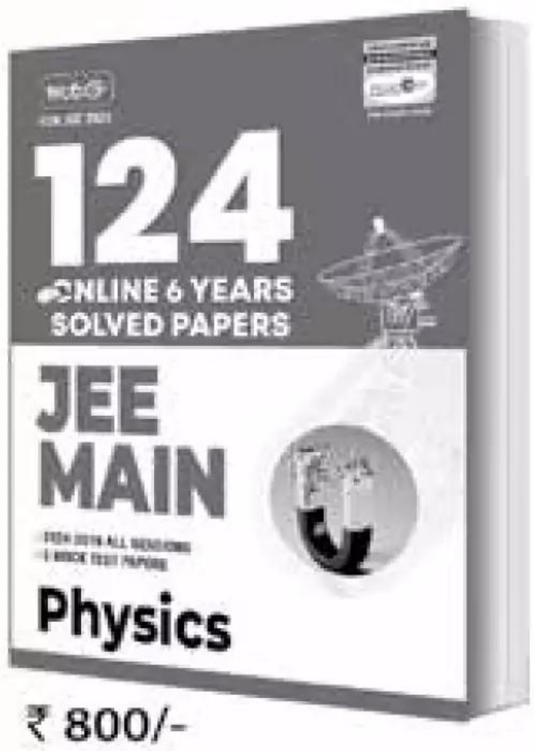

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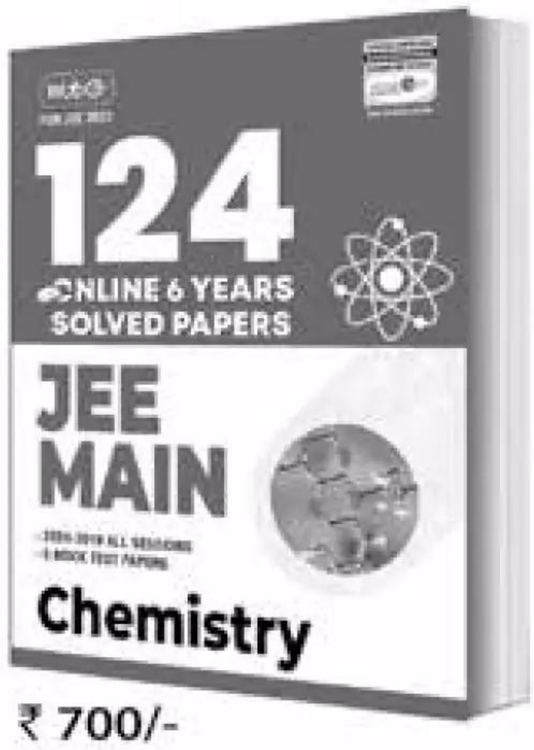

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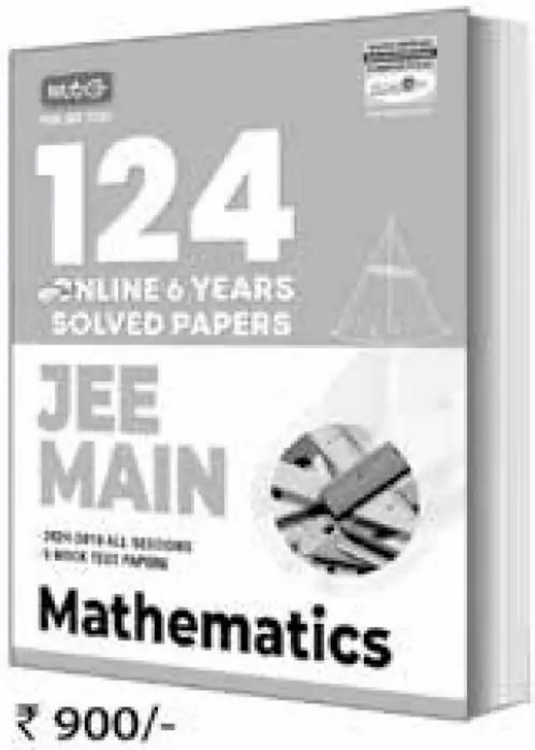

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SOLVED PAPER

Held on
22nd Jan, Morning Shift
(Session - 1)

JEE MAIN 2025

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. Let $x = x(y)$ be the solution of the differential equation $y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If $x(1)=1$, then $x\left(\frac{1}{2}\right)$ is :

(a) $3+e$ (b) $\frac{3}{2}+e$ (c) $\frac{1}{2}+e$ (d) $3-e$

2. Let $A=\{1, 2, 3,...., 10\}$ and

$B=\left\{\frac{m}{n}: m, n\in A, m<n \text{ and } \gcd(m, n)=1\right\}$. Then $n(B)$ is equal to :

(a) 36 (b) 31 (c) 37 (d) 29

3. If $\sum_{r=1}^{n}T_r=\frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$,

then $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{T_r}\right)$ is equal to:

(a) 0 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) 1

4. The area of the region, inside the circle $\left(x-2\sqrt{3}\right)^2+y^2=12$ and outside the parabola $y^2=2\sqrt{3}x$ is :

(a) $3\pi+8$ (b) $3\pi-8$
(c) $6\pi-16$ (d) $6\pi-8$

5. The product of all solutions of the equation $e^{5(\log_e x)^2+3}=x^8, x>0$, is :

(a) e^2 (b) e (c) $e^{8/5}$ (d) $e^{6/5}$

6. Let $L_1:\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and

$L_2:\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2?

(a) $\left(-\frac{5}{3},-7,1\right)$ (b) $\left(2,3,\frac{1}{3}\right)$

(c) $\left(\frac{14}{3},-3,\frac{22}{3}\right)$ (d) $\left(\frac{8}{3},-1,\frac{1}{3}\right)$

7. Let $a_1, a_2, a_3, ...$ be a G.P. of increasing positive terms. If $a_1a_5=28$ and $a_2+a_4=29$, then a_6 is equal to:

(a) 812 (b) 526 (c) 784 (d) 628

8. Let for $f(x)=7\tan^8x+7\tan^6x-3\tan^4x-3\tan^2x$,

$I_1=\int_0^{\pi/4}f(x)dx$ and $I_2=\int_0^{\pi/4}xf(x)dx$. Then $7I_1+12I_2$ is equal to :

(a) 2 (b) π (c) 1 (d) 2π

9. Let the foci of a hyperbola be (1, 14) and (1, –12). If it passes through the point (1, 6) then the length of its latus-rectum is :

(a) $\frac{25}{6}$ (b) $\frac{24}{5}$ (c) $\frac{144}{5}$ (d) $\frac{288}{5}$

10. From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is 'M', is :

(a) 14950 (b) 5148 (c) 4356 (d) 6084

11. The number of non-empty equivalence relations on the set {1, 2, 3} is:

(a) 5 (b) 6 (c) 7 (d) 4

12. Let $f(x)$ be a real differentiable function such that $f(0)=1$ and $f(x+y)=f(x)f'(y)+f'(x)f(y)$ for all $x, y\in\mathbf{R}$.

Then $\sum_{n=1}^{100}\log_e f(n)$ is equal to :

(a) 2406 (b) 5220 (c) 2525 (d) 2384

13. Using the principal values of the inverse trigonometric functions, the sum of the maximum and the minimum values of $16((\sec^{-1}x)^2+(\text{cosec}^{-1}x)^2)$ is :

(a) $22\pi^2$ (b) $24\pi^2$ (c) $18\pi^2$ (d) $31\pi^2$

14. A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point (2, 5) and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α, β), then $3\beta - 2\alpha$ is equal to :

(a) 12 (b) 14 (c) 10 (d) 15

15. Let z_1, z_2 and z_3 be three complex number on the circle $|z| = 1$ with $\arg(z_1) = \frac{-\pi}{4}$, $\arg(z_2) = 0$ and $\arg(z_3) = \frac{\pi}{4}$. If $|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2 = \alpha + \beta\sqrt{2}$, $\alpha, \beta \in \mathbf{Z}$, then the value of $\alpha^2 + \beta^2$ is :

(a) 24 (b) 41 (c) 29 (d) 31

16. Let the triangle PQR be the image of the triangle with vertices (1, 3), (3, 1) and (2, 4) in the line $x + 2y = 2$. If the centroid of ΔPQR is the point (α, β), then $15(\alpha - \beta)$ is equal to:

(a) 19 (b) 24 (c) 21 (d) 22

17. Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected balls is black, given that the second selected ball is also black is $\frac{m}{n}$, where $gcd(m, n) = 1$, then $m + n$ is equal to :

(a) 11 (b) 13 (c) 4 (d) 14

18. A coin is tossed three times. Let X denote the number of times a tail follows a head. If μ and σ^2 denote the mean and variance of X, then the value of $64(\mu + \sigma^2)$ is :

(a) 64 (b) 48 (c) 51 (d) 32

19. Let the parabola $y = x^2 + px - 3$, meet the coordinate axes at the points P, Q and R. If the circle C with centre at (–1, –1) passes through the points P, Q and R, then the area of ΔPQR is :

(a) 7 (b) 4 (c) 6 (d) 5

20. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that $f(x + y) = f(x)\,f(y)$ for all $x, y \in \mathbb{R}$. If $f'(0) = 4a$ and f satisfies $f''(x) = 3a\,f'(x) - f(x) = 0$, $a > 0$, then the area of the region $R = \{(x, y) \mid 0 \le y \le f(ax), 0 \le x \le 2\}$ is :

(a) $e^4 + 1$ (b) $e^2 - 1$ (c) $e^4 - 1$ (d) $e^2 + 1$

SECTION-B (NUMERICAL VALUE TYPE)

21. If $\sum_{r=0}^{5} \frac{^{11}C_{2r+1}}{2r+2} = \frac{m}{n}$, $gcd\,(m, n) = 1$, then $m - n$ is equal to _____.

22. Let the function, $f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ a^2 + bx, & x \ge 1 \end{cases}$ be differentiable for all $x \in \mathbb{R}$, wehe $a > 1$, $b \in \mathbb{R}$. If the area of the region enclosed by $y = f(x)$ and the line $y = -20$ is $\alpha + \beta\sqrt{3}$, $\alpha, \beta \in \mathbf{Z}$, then the value of $\alpha + \beta$ is _____.

23. Let A be a square matrix of order 3 such that $\det(A) = -2$ and $\det(3\text{adj}(-6\text{adj}(3A))) = 2^{m+n} \cdot 3^{mn}$, $m > n$. Then $4m + 2n$ is equal to _____.

24. Let $L_1 : \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and

$L_2 : \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{\alpha}$, $\alpha \in \mathbb{R}$, be two lines, which intersect at the point B. If P is the foot of perpendicular from the point $A(1, 1, -1)$ and L_2, then the value of 26α $(PB)^2$ is _____.

25. Let $\vec{c}$ be the projection vector of $\vec{b} = \lambda\hat{i} + 4\hat{k}$, $\lambda > 0$, on the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$. If $|\vec{a} + \vec{c}| = 7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is _____.

SOLUTIONS

1. (d) : We have, $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$

$\Rightarrow y^2dx = \left(\frac{1}{y} - x\right)dy \quad \Rightarrow \frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$,

which is a linear D.E with $P = \frac{1}{y^2}$ and $Q = \frac{1}{y^3}$

$\text{I.F} = e^{\int P dy} = e^{\int \frac{1}{y^2}dy} = e^{\frac{-1}{y}}$

$\therefore$ Solution is given by

$x \cdot e^{-1/y} = \int \frac{1}{y^3} \cdot e^{-1/y}dy + c$

Putting $\frac{1}{y} = t \quad \Rightarrow \frac{-1}{y^2}dy = dt$

$\Rightarrow x \cdot e^{\frac{-1}{y}} = -\int t \cdot e^{-t}dy + c$

$\Rightarrow x \cdot e^{\frac{-1}{y}} = -e^{-t}(-t - 1) + c$

$\Rightarrow x \cdot e^{\frac{-1}{y}} = e^{\frac{-1}{y}}\left(1 + \frac{1}{y}\right) + c \Rightarrow x = 1 + \frac{1}{y} + ce^{1/y}$

Now, at $x(1) = 1$, *i.e.*, when $x = 1$, $y = 1$

$\Rightarrow 1 = 1 + 1 + ce \quad \Rightarrow c = \frac{-1}{e}$

$\therefore$ Solution is, $x = 1 + \frac{1}{y} - \frac{1}{e}\left(e^{\frac{1}{y}}\right)$

$\therefore \quad x\left(\frac{1}{2}\right) = 1 + 2 - \frac{1}{e} \times e^2 = 3 - e$

2. (b) : We have, $A = \{1, 2, 3, ..., 10\}$

$$B = \left\{\frac{m}{n} : m, n \in A, m < n \text{ and } \gcd(m, n) = 1\right\}$$

For $m = 1$, $n = 2, 3, ..., 10 \Rightarrow$ 9 cases
$m = 2$, $n = 3, 5, 7, 9, \Rightarrow$ 4 cases
$m = 3$, $n = 4, 5, 7, 8, 10 \Rightarrow$ 5 cases
$m = 4$, $n = 5, 7, 9 \Rightarrow$ 3 cases
$m = 5$, $n = 6, 7, 8, 9 \Rightarrow$ 4 cases
$m = 6$, $n = 7 \Rightarrow$ 1 case
$m = 7$, $n = 8, 9, 10 \Rightarrow$ 3 cases
$m = 8$, $n = 9 \Rightarrow$ 1 case
$m = 9$, $n = 10 \Rightarrow$ 1 case

$\therefore \quad n(B) = 31$

3. (b) : Given,

$$S_n = \sum_{r=1}^{n} T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$$

So, $S_{n-1} = \dfrac{(2n-3)(2n-1)(2n+1)(2n+3)}{64}$

$\because \quad T_n = S_n - S_{n-1}$

$$\Rightarrow T_n = \frac{(2n-1)(2n+1)(2n+3)}{8}$$

$$\therefore \quad \sum_{r=1}^{n}\left(\frac{1}{T_n}\right) = \frac{8}{4}\left[\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right]$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{T_r}\right) = 2\lim_{n\to\infty}\left(\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right)$$

$$\Rightarrow \lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{T_r}\right) = 2\left(\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{3\cdot 5} - \frac{1}{5\cdot 7} + ...\right)$$

$$= \lim_{n\to\infty}\frac{2}{3} - \frac{2}{(2n+1)(2n+3)} = \frac{2}{3}$$

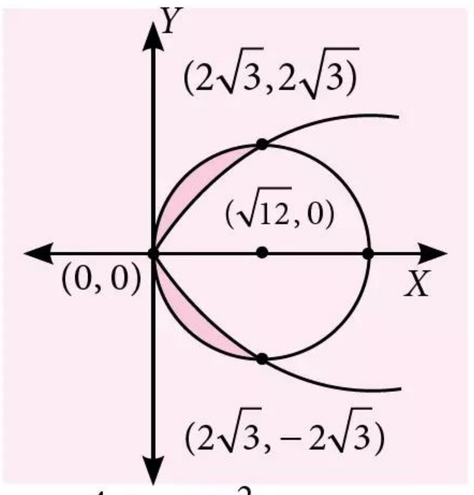
4. (c) : We have, $\left(x - 2\sqrt{3}\right)^2 + y^2 = 12$, a circle with centre $\left(2\sqrt{3}, 0\right)$ and radius = $\sqrt{12}$ and a parabola $y^2 = 2\sqrt{3}x$.

$\therefore$ Required area

$$= \frac{\pi(12)}{2} - 2\int_0^{2\sqrt{3}} \left(2\sqrt{3}\right)^{\frac{1}{2}} x^{1/2}\,dx$$

$$= 6\pi - 2\left(2\sqrt{3}\right)^{1/2}\left[\frac{x^{3/2}}{3}\cdot 2\right]_0^{2\sqrt{3}}$$

$$= 6\pi - \frac{4}{3}\left(2\sqrt{3}\right)^{1/2}\left(2\sqrt{3}\right)^{3/2} = 6\pi - \frac{4}{3}\left(2\sqrt{3}\right)^2 = 6\pi - 16$$



5. (c) : We have, $e^{5(\log e x)^2 + 3} = x^8$, $x > 0$

Taking log on both sides, we get

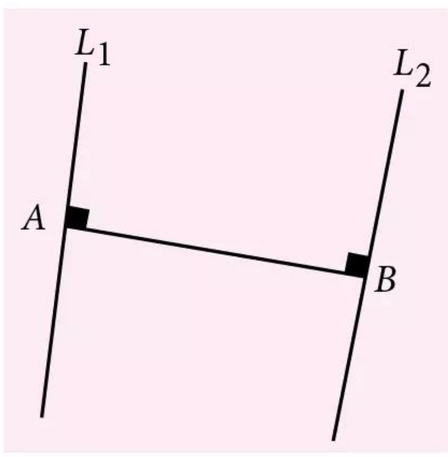
$5(\ln x)^2 + 3 = 8 \ln x \Rightarrow 5(\ln x)^2 + 3 - 8 \ln x = 0$

Put $\ln x = t \Rightarrow 5t^2 - 8t + 3 = 0 \Rightarrow t_1 + t_2 = \dfrac{8}{5}$

$\therefore \quad \ln x_1 + \ln x_2 = 8/5 \Rightarrow \ln(x_1 x_2) = 8/5 \Rightarrow x_1 x_2 = e^{8/5}$

6. (c) : We have, $L_1 : \dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$

and $L_2 : \dfrac{x-2}{3} = \dfrac{y-4}{4} = \dfrac{z-5}{5}$



Let $A\,(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$ and $B(3\mu + 2, 4\mu + 4, 5\mu + 5)$ lies on the line L_1 and L_2, respectively.

Direction ratios of AB is
$< 3\mu - 2\lambda + 1, 4\mu - 3\lambda + 2, 5\mu - 4\lambda + 2 >$

As $AB \perp L_1$, we have

$2\,(3\mu - 2\lambda + 1) + 3\,(4\mu - 3\lambda + 2) + 4\,(5\mu - 4\lambda + 2) = 0$

$\Rightarrow \quad 6\mu - 4\lambda + 2 + 12\mu - 9\lambda + 6 + 20\mu - 16\lambda + 8 = 0$

$\Rightarrow \quad 38\mu - 29\lambda + 16 = 0 \qquad ....(i)$

Also, $AB \perp L_2$, we have

$3\,(3\mu - 2\lambda + 1) + 4\,(4\mu - 3\lambda + 2) + 5\,(5\mu - 4\lambda + 2) = 0$

$\Rightarrow \quad 50\mu - 38\lambda + 21 = 0 \qquad ....(ii)$

Solving (i) and (ii), we get

$$\lambda = \frac{1}{3} \text{ and } \mu = \frac{-1}{6}$$

$\therefore$ Coordinate of A is $\left(\dfrac{5}{3}, 3, \dfrac{13}{3}\right)$ and $B\left(\dfrac{3}{2}, \dfrac{10}{3}, \dfrac{25}{6}\right)$

Equation of AB is given by

$$\frac{x - \frac{5}{3}}{\frac{1}{6}} = \frac{y-3}{-\frac{1}{3}} = \frac{z - \frac{13}{3}}{\frac{1}{6}} \Rightarrow x - \frac{5}{3} = \frac{y-3}{-2} = z - \frac{13}{3}$$

$\therefore$ Point $\left(\dfrac{14}{3}, -3, \dfrac{22}{3}\right)$ lies on line AB.

7. (c) : Here, $r > 0$ and $a_1, a_2, a_3, ... > 0$ as G.P. has increasing positive terms.

$a_1 a_5 = 28$ (Given)

$\Rightarrow \quad a(ar^4) = 28 = a^2r^4 = 28 \qquad ...(i)$

Also, $a_2 + a_4 = 29 \quad \Rightarrow \quad ar + ar^3 = 29$

$\Rightarrow \quad ar\,(1 + r^2) = 29 \qquad ...(ii)$

From (i) and (ii), we get,

$$\frac{r^2}{(1+r^2)^2} = \frac{28}{(29)^2} \Rightarrow r^2 = 28 \text{ or } \frac{1}{28}$$

$$\Rightarrow \quad r = \sqrt{28} \qquad \left[\because r \neq \frac{1}{28} \text{ as G.P. is increasing}\right]$$

$$\Rightarrow \quad a = \frac{1}{\sqrt{28}} \text{ and } a_6 = ar^5 = \frac{1}{\sqrt{28}} \times \left(\sqrt{28}\right)^5 = 784$$

8. (c) : We have, $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$

$= 7\tan^6 x\,(\tan^2 x + 1) - 3\tan^2 x\,(\tan^2 x + 1)$

$= (7\tan^6 x - 3\tan^2 x)\sec^2 x$

$$I_1 = \int_0^{\pi/4} \left(7\tan^6 x - 3\tan^2 x\right)\sec^2 x\, dx$$

Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$I_1 = \int_0^1 \left(7t^6 - 3t^2\right)dt = \left[t^7 - t^3\right]_0^1 = 0$$

$$I_2 = \int_0^{\pi/4} x\left(7\tan^6 x - 3\tan^2 x\right)\sec^2 x\, dx$$

$$= \int_0^1 \left(7t^6 - 3t^2\right)\tan^{-1} t\, dt$$

$$= \left[\tan^{-1} t\left(t^7 - t^3\right)\right]_0^1 - \int_0^1 \frac{t^7 - t^3}{1+t^2}dt = 0 - \int_0^1 \frac{t^3(t^4-1)}{1+t^2}dt$$

$$= -\int_0^1 t^3(t^2-1)dt = -\int_0^1 (t^5 - t^3)dt$$

$$= -\left[\frac{t^6}{6} - \frac{t^4}{4}\right]_0^1 = \frac{1}{12}$$

$\therefore \quad 7I_1 + 12I_2 = 0 + 12\left(\frac{1}{12}\right) = 1$

9. (d) : Distance between foci $= \sqrt{(1-1)^2 + (14+12)^2}$

$\Rightarrow \quad 2be = 26 \Rightarrow be = 13$

Mid-point of foci $= = \left(\frac{1+1}{2}, \frac{14-12}{2}\right) = (1, 1)$

Equation of hyperbola is $\frac{(x-1)^2}{a^2} + \frac{(y-1)^2}{b^2} = 1$

Hyperbola passes through (1, 6)

$$\Rightarrow \frac{(6-1)^2}{b^2} = 1 \qquad \Rightarrow \quad b^2 = 25$$

Now, $a^2 = b^2(e^2 - 1) = b^2e^2 - b^2 = 169 - 25 = 144$

$\therefore \quad a^2 = 144$ and $b^2 = 25$

$\therefore$ Length of latus rectum $= \frac{2a^2}{b} = \frac{2\times144}{5} = \frac{288}{5}$

10. (b) : Number of English alphabets before M = 12

Number of English alphabets after M = 13

$\therefore$ Required number of ways $= {}^{12}C_2 \times {}^{13}C_2$

$= 66 \times 78 = 5148$

11. (a) : Partitions of set {1, 2, 3} is {{1}}, {2}, {3}}, {{1, 2}, {3}}, {{1, 3}, {2}}, {{2, 3}, {1}}, {{1, 2, 3}}

$\therefore$ Number of non-empty equivalence relations on the set {1, 2, 3} = 5

12. (c) : $f(x + y) = f(x)f'(y) + f'(x)f(y)$

When $x = 0, y = 0$, we have

$f(0) = f(0)f'(0) + f'(0)f(0)$

$\Rightarrow \quad f(0) = 2f'(0)f(0) \Rightarrow f'(0) = \frac{1}{2}$

When, $y = 0, f(x) = f(x)f'(0) + f(0)f'(x)$

$\Rightarrow \frac{1}{2}f(x) = f'(x) \qquad [\because \ f(0) = 1]$

Integrating both sides, we get

$f(x) = e^{x/2} \cdot C$

Now, $f(0) = 1 \quad \Rightarrow \quad f(x) = e^{x/2} \quad \Rightarrow \quad \log_e f(x) = \frac{x}{2}$

$$\therefore \quad \sum_{n=1}^{100} \log_e f(n) = \sum_{n=1}^{100} \frac{n}{2} = \frac{1}{2}\frac{(100)(101)}{2} = 2525$$

13. (a) : $f(x) = 16((\sec^{-1} x)^2 + (\operatorname{cosec}^{-1} x)^2)$

$= 16\left[(\sec^{-1} x + \operatorname{cosec}^{-1} x)^2 - 2\sec^{-1} x \operatorname{cosec}^{-1} x\right]$

$$= 16\left[\left(\frac{\pi}{2}\right)^2 - 2\left(\frac{\pi}{2} - \operatorname{cosec}^{-1} x\right)\operatorname{cosec}^{-1} x\right]$$

$$= 16\left[\frac{\pi^2}{4} - \pi\operatorname{cosec}^{-1} x + 2(\operatorname{cosec}^{-1} x)^2\right]$$

$$= 32\left[(\operatorname{cosec}^{-1} x)^2 - \frac{\pi}{2}\operatorname{cosec}^{-1} x + \frac{\pi^2}{8}\right]$$

$$\Rightarrow \quad f(x) = 32\left[\left(\operatorname{cosec}^{-1} x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$$

$f(x)$ is maximum when $x = -1$

$$\therefore \quad f_{max} = 32 \times 10\frac{\pi^2}{16}$$

$f(x)$ is minimum when $x = \sqrt{2}$

$$\therefore \quad f_{min} = 32 \times \frac{\pi^2}{16}$$

$$\therefore \quad \text{Required sum} = 32 \times 10\frac{\pi^2}{16} + 32 \times \frac{\pi^2}{16} = 22\pi^2$$

14. (d) : Circle with radius r touches the circle C, when $r + 2$ = distance between their centres

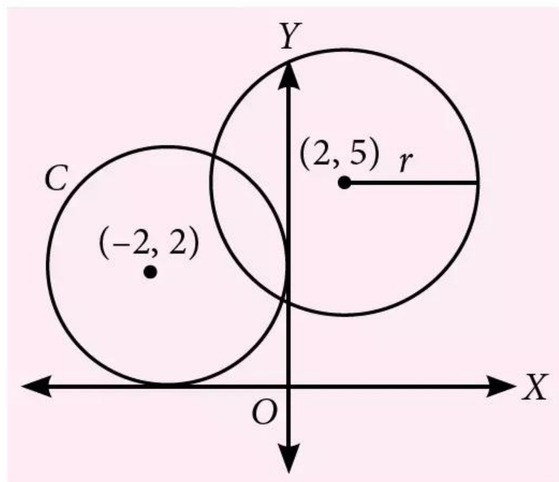
i.e., $r + 2 = \sqrt{4^2 + 3^2} = 5$

Also, if circle C touches the circle with radius r internally,

then $r = 2 + \sqrt{4^2 + 3^2}$

$= 2 + 5 = 7$

Since, circle with radius r intersects the circle C at exactly 2 points.



$\therefore \quad r + 2 > 5$ and $r < 7$

i.e., $3 < r < 7$

$\therefore \quad \alpha = 3, \beta = 7$

$\Rightarrow \quad 3\beta - 2\alpha = (3)(7) - (2)(3) = 21 - 6 = 15$

15. (c) : Given, $|z| = 1$

Now, $z_1 = e^{i\frac{\pi}{4}} = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} = \frac{1-i}{\sqrt{2}}$; $z_2 = e^{i0} = 1$;

$$z_3 = e^{i\frac{\pi}{4}} = \frac{1+i}{\sqrt{2}}$$

Now, $\left|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1\right|^2 = \left|\frac{1-i}{\sqrt{2}}\cdot 1 + 1\cdot\frac{1-i}{\sqrt{2}} + \left(\frac{1+i}{2}\right)^2\right|^2$

$= \left|\sqrt{2} + (1-\sqrt{2})i\right|^2 = 5 - 2\sqrt{2} = \alpha + \beta\sqrt{2}$

$\Rightarrow \alpha = 5$ and $\beta = -2 \quad \therefore \alpha^2 + \beta^2 = 29$

16. (d) : Let P, Q and R be the mirror images of points (1,3), (3, 1) and (2, 4) in the line $x + 2y - 2 = 0$.

Coordinates of P is given by

$$\frac{x-1}{1} = \frac{y-3}{2} = -2\left(\frac{1+6-2}{5}\right)$$

i.e., $P(-1, -1)$ Similarly $Q\left(\frac{9}{5}, \frac{-7}{5}\right)$ and $R\left(\frac{-6}{5}, \frac{-12}{5}\right)$

$$\text{Centroid of } \Delta PQR = \left(\frac{-2}{15}, -\frac{8}{5}\right) \therefore \alpha = \frac{-2}{15} \text{ and } \beta = -\frac{-8}{5}$$

Thus, $15(\alpha - \beta) = 15\left(\frac{-2}{15} + \frac{8}{5}\right) = 22$

17. (d) : Required probability

$$= \frac{\frac{6}{10}\times\frac{5}{9}}{\frac{4}{10}\times\frac{6}{9} + \frac{6}{10}\times\frac{5}{9}} = \frac{30}{24+30} = \frac{30}{54} = \frac{5}{9}$$

So, $m = 5, n = 9 \quad [\because gcd(m, n) = 1]$

$\therefore \quad m + n = 14$

18. (b) : When a coin is tossed three times, outcomes are : {HHH, HHT, HTH, THH, THT, TTH, HTT and TTT}

Outcomes when tails does not follows a head (only once): {HHH, THH, TTT, TTH}

Outcomes when tail follows a head :

{HHT, HTT, HTH, THT}.

$\therefore$ Probability distribution table is given by

X_i	0	1
$P(X_i)$	1/2	1/2

$$\text{Mean } (\mu) = \sum X_i P_i = \frac{1}{2}$$

$$\text{and variance } (\sigma^2) = \sum X_i^2 P_i - \mu^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$\therefore \quad 64(\mu + \sigma^2) = 64\left(\frac{1}{2} + \frac{1}{4}\right) = 64 \times \frac{3}{4} = 48$$

19. (c) : Given, equation of parabola is $y = x^2 + px - 3$ and equation of circle with centre at $(-1, -1)$ is

$(x + 1)^2 + (y + 1)^2 = r^2$ ...(i)

Let $P(\alpha, 0)$, $Q(\beta, 0)$ and $R(0, -3)$ are the three points.

Now, circle passes through point $R(0, -3)$

$\Rightarrow \quad 1 + (-2)^2 = r^2 \Rightarrow 5 = r^2$

Put $y = 0$ in equation (i), we get

$(x + 1)^2 + 1 = 5$

$\Rightarrow \quad (x + 1)^2 = 4 \quad \Rightarrow \quad x = 1$ or $x = -3$

$\Rightarrow$ Point are $P(1, 0)$ and $(-3, 0)$

$$\therefore \quad \text{Area of } \Delta PQR = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ -3 & 0 & 1 \\ 0 & -3 & 1 \end{vmatrix} = 6 \text{ sq. units}$$

20. (b)

21. (2035) : We know that,

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots\ldots\ldots + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{(n+1)} \quad \ldots(i)$$

$$C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \ldots\ldots\ldots + (-1)^n\frac{C_n}{n+1} = \frac{1}{n+1} \quad \ldots(ii)$$

Subtracting (ii) from (i), we get

$$2\left[\frac{C_1}{2} + \frac{C_3}{4} + \ldots\ldots\ldots\right] = \frac{2^{n+1}-1}{(n+1)} - \frac{1}{(n+1)}$$

Put $n = 11$, we get

$$2\left[\frac{C_1}{2} + \frac{C_3}{4} + \ldots\ldots\ldots + \frac{C_{11}}{12}\right] = \frac{2^{12}-1}{12} - \frac{1}{12}$$

$$\Rightarrow \quad \frac{C_1}{2} + \frac{C_3}{4} + \ldots\ldots\ldots + \frac{C_{11}}{12} = \frac{1}{2}\left[\frac{2^{12}-2}{12}\right] \quad \ldots(iii)$$

Now, $\sum_{r=0}^{5} \frac{^{11}C_{2r+1}}{2r+2} = \frac{m}{n}$

$$\Rightarrow \quad \frac{^{11}C_1}{2} + \frac{^{11}C_3}{4} + \frac{^{11}C_5}{6} + \ldots\ldots\ldots + \frac{^{11}C_{11}}{12} = \frac{m}{n}$$

$$\Rightarrow \quad \frac{1}{2}\left[\frac{2^{12}-2}{12}\right] = \frac{m}{n} \qquad \text{[From (iii)]}$$

$$\Rightarrow \quad \frac{2^{11}-1}{12} = \frac{m}{n} \quad \Rightarrow \quad \frac{m}{n} = \frac{2047}{12}$$

$\therefore \quad m - n = 2047 - 12 = 2035$

22. (34) : Given, $f(x)$ is continuous and differentiable at $x = 1$.

$$f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ a^2 + bx, & x \geq 1 \end{cases}$$

$$\Rightarrow \quad f'(x) = \begin{cases} -6ax, & x < 1 \\ b, & x \geq 1 \end{cases}$$

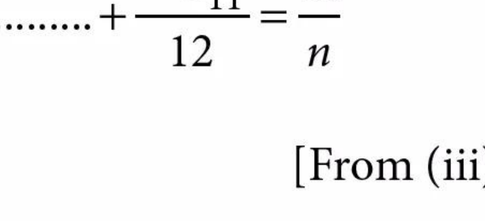
L.H.L at $x = 1 \quad \Rightarrow -3a - 2$

R.H.L at $x = 1 \quad \Rightarrow a^2 + b$

$\therefore \quad$ L.H.L = R.H.L. $\{\because f(x)$ is continuous$\}$

$\Rightarrow \quad -3a - 2 = a^2 + b$ ... (i)

L.H.D at $x = 1 \quad \Rightarrow -6a$



R.H.D at $x = 1 \Rightarrow b$

$\therefore$ L.H.D = R.H.D [$\because$ $f(x)$ is differentiable]

$\Rightarrow -6a = b$ ...(ii)

From (i) and (ii), we get

$-3a - 2 = a^2 - 6a$

$\Rightarrow a^2 - 3a + 2 = 0 \Rightarrow (a-1)(a-2) = 0$

$\Rightarrow a = 1$ or $a = 2 \Rightarrow a = 2$ ($\because$ $a > 1$)

From (ii), $b = -12$

Now, $f(x) = \begin{cases} -6x^2 - 2, & x < 1 \\ 4 - 12x, & x \geq 1 \end{cases}$

$\therefore$ Area of region

$$= \int_{-\sqrt{3}}^{1} \left(-6x^2 - 2 + 20\right) dx + \int_{1}^{2} (4 - 12x + 20)\, dx$$

$$= \left(\frac{-6x^3}{3} - 2x + 20x\right)_{-\sqrt{3}}^{1} + \left(4x - \frac{12x^2}{2} + 20x\right)_{1}^{2}$$

$\Rightarrow = 16 + 12\sqrt{3} + 6 = 22 + 12\sqrt{3}$

$\Rightarrow 22 + 12\sqrt{3} = \alpha + \beta\sqrt{3}$

So, $\therefore \alpha = 22$ and $\beta = 12$

$\therefore \alpha + \beta = 22 + 12 = 34$

23. (34) : We have, $|A| = -2$

and $\det(3 \text{ adj } (-6 \text{ adj}(3A))) = 2^{m+n} \cdot 3^{mn}$

$\Rightarrow 3^3 \det(\text{adj } (-6 \text{ adj}(3A))) = 2^{m+n} \cdot 3^{mn}$

$\Rightarrow 3^3 |-6 \text{ adj } (3A)|^2 = 2^{m+n} \cdot 3^{mn}$

$\Rightarrow 3^3 ((-6)^3)^2 \,|\, \text{adj } (3A)|^2 = 2^{m+n} \cdot 3^{mn}$

$\Rightarrow 3^3 (6)^6 \,|\, (3A)^2|^2 = 2^{m+n} \cdot 3^{mn}$

$\Rightarrow 3^3 \times 6^6 \times (3^3)^4 |A|^4 = 2^{m+n} \cdot 3^{mn}$

$\Rightarrow 3^{21} \times 2^6 \times (-2)^4 = 2^{m+n} \cdot 3^{mn}$

$\Rightarrow 3^{21} \times 2^{10} = 2^{m+n} \cdot 3^{mn}$

On comparing the powers, we get

$m + n = 10$ and $mn = 21$

$\therefore m = 7$ and $n = 3$

$\therefore$ Value of $4m + 2n = 4 \times 7 + 2 \times 3 = 34$

24. (216) : Point $B \equiv (3\mu + 1, -\mu + 1, -1)$

$\equiv (2\lambda + 2.., 0, \alpha\lambda - 4)$

$\Rightarrow 3\mu + 1 = 2\lambda + 2, -\mu + \lambda = 0, \lambda\alpha - 4 = -1$

$\therefore \mu = 1 \Rightarrow \lambda = 1$ and $\alpha = 3$

So, point $B(4, 0, -1)$.

Let point P is $(2k + 2, 0, 3k - 4)$.

So, Dr's of AP is $< 2k + 1, -1, 3k - 3 >$

Since, $L_2 \perp AP$

$\Rightarrow 2(2k+1) + 0(-1) + 3(3k-3) = 0$

$\Rightarrow 4k + 2 + 9k - 9 = 0 \Rightarrow k = \frac{7}{13}$

$\therefore$ Point $P \equiv \left(\frac{40}{13}, 0, \frac{-31}{13}\right)$

$$\therefore (PB)^2 = \left(\frac{40}{13} - 4\right)^2 + (0-0)^2 + \left(\frac{-31}{13} + 1\right)^2 = \frac{468}{169}$$

$$\therefore 26\alpha(PB)^2 = 26 \times 3 \times \frac{468}{169} = 216$$

25. (16) : Here, $\vec{c} = \left(\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|} = \left(\frac{\lambda + 8}{9}\right)\vec{a}$

Now, $|\vec{a} + \vec{c}| = \left|\vec{a} + \left(\frac{\lambda + 8}{9}\right)\vec{a}\right| = 7$ [Given]

$\Rightarrow |\vec{a}|\left|\frac{\lambda + 17}{9}\right| = 7 \Rightarrow \left|\frac{\lambda + 17}{9}\right| \times 3 = 7 \Rightarrow \left|\frac{\lambda + 17}{3}\right| = 7$

$\Rightarrow |\lambda + 17| = 21$

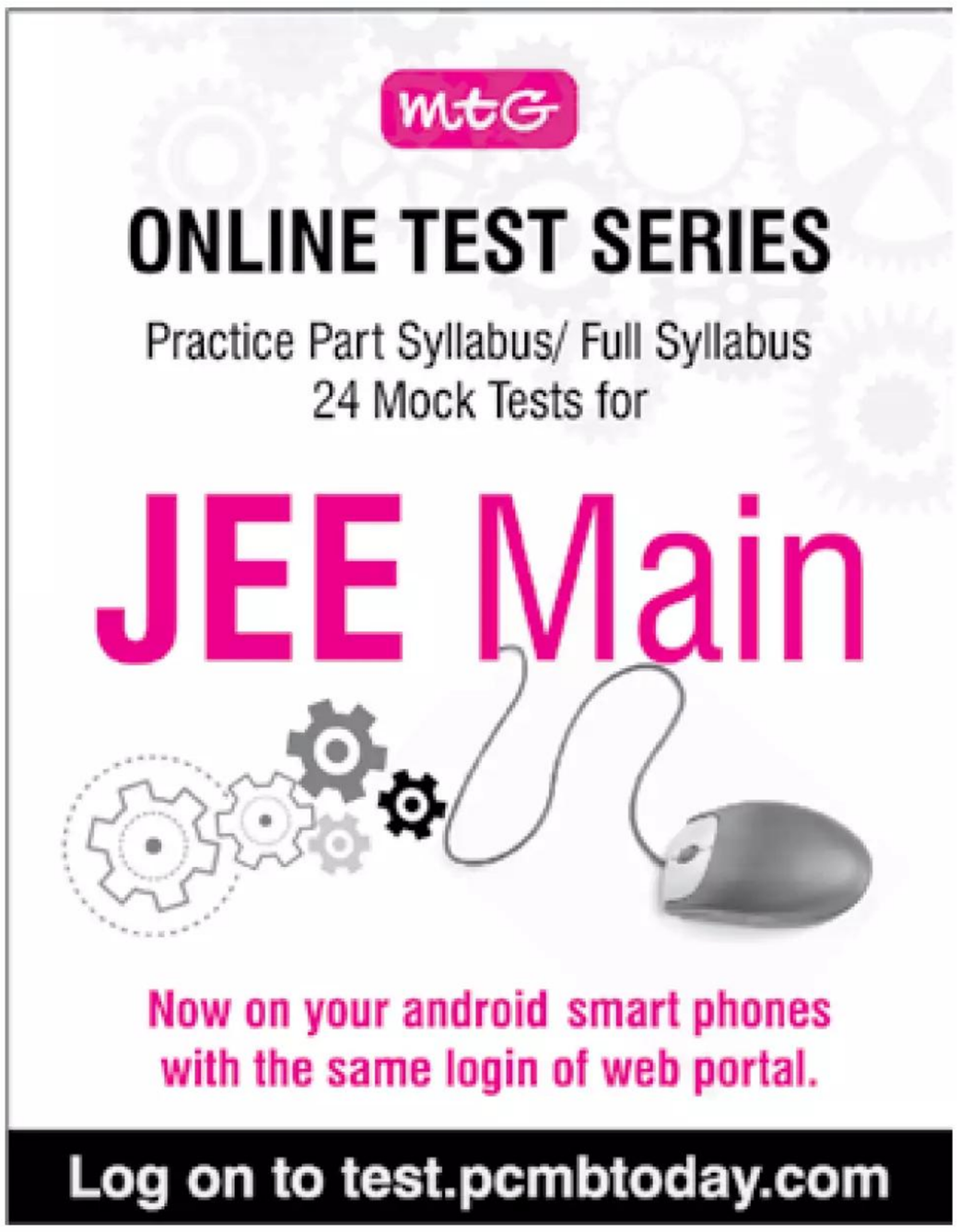
Since, $\lambda > 0 \Rightarrow \lambda = 4$

$\therefore \vec{c} = \frac{4}{3}\vec{a} = \frac{4}{3}(\hat{i} + 2\hat{j} + 2\hat{k}), \vec{b} = 4(\hat{i} + \hat{k})$

Now, $\vec{b} \times \vec{c} = \frac{16}{3}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 1 & 2 & 2 \end{vmatrix} = \frac{16}{3}(-2\hat{i} - \hat{j} + 2\hat{k})$

$\therefore$ Area of parallelogram $= |\vec{b} \times \vec{c}| = \frac{16}{3} \times 3$

$= 16$ sq. units



JEE WORK GUTS

SINGLE OPTION CORRECT TYPE

1. The coordinates of the point P on the line $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(-\hat{i} + \hat{j} - \hat{k})$ which is nearest to the origin is

(a) $\left(\frac{2}{3}, \frac{4}{3}, \frac{2}{3}\right)$ (b) $\left(-\frac{2}{3}, -\frac{4}{3}, \frac{2}{3}\right)$

(c) $\left(\frac{2}{3}, \frac{4}{3}, -\frac{2}{3}\right)$ (d) None of these

2. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1, the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by

(a) $6 + 7i$ (b) $-7 + 6i$

(c) $7 + 6i$ (d) $-6 + 7i$

3. If the system of equations $2x - y + z = 0$, $x - 2y + z = 0$, $tx - y + 2z = 0$ has infinitely many solutions and $f(x)$ be a continuous function such that $f(5 + x) + f(x) = 2$, then $\int_0^{-2t} f(x)\,dx$ is equal to

(a) 0 (b) $-2t$ (c) 5 (d) t

4. An artillery target may be either at point I with probability 8/9 or at point II with probability 1/9. We have 55 shells, each of which can be fired either at point I or II. Each shell may hit the target, independent of the other shells, with probability 1/2. Maximum number of shells must be fired at point I to have maximum probability is

(a) 20 (b) 25 (c) 29 (d) 35

5. If $U_n = \sin n\theta \cdot \sec^n \theta$, $V_n = \cos n\theta \cdot \sec^n \theta$ for $n = 0$, 1, 2, ..., then $V_n - V_{n-1} + U_{n-1} \tan \theta$ is equal to

(a) 0 (b) 1 (c) $\sin \theta$ (d) $\cos \theta$

6. If $f(n) = \frac{1}{n}[(n + 1)(n + 2)(n + 3) \ldots (n + n)]^{1/n}$, then $\lim_{n \to \infty} f(n)$ equals

(a) e (b) $1/e$ (c) $2/e$ (d) $4/e$

7. Let $f : A \to A$ be an invertible function, where $A = \{1, 2, 3, 4, 5, 6\}$. The number of these functions in which at least three elements have self-image is

(a) 36 (b) 42 (c) 56 (d) 60

8. Let $f(x) = \alpha(x)\,\beta(x)\,\gamma(x)$ for all real x, where $\alpha(x)$, $\beta(x)$ and $\gamma(x)$ are differentiable functions of x. If $f'(2) = 18\,f(2)$, $\alpha'(2) = 3\alpha(2)$, $\beta'(2) = -4\beta(2)$ and $\gamma'(2) = k\,\gamma(2)$, then the value of k is

(a) 14 (b) 16

(c) 19 (d) None of these

9. $OPQR$ is a square and M, N are the middle points of the sides PQ and QR respectively, then the ratio of the areas of the square and the triangle OMN is

(a) 4 : 1 (b) 2 : 1 (c) 8 : 3 (d) 4 : 3

10. Four squares are randomly selected from a chess board. If the probability that squares lies on the diagonal line is, $\frac{m}{C(64, 4)}$, then the value of m equals

(a) 112 (b) 224 (c) 182 (d) 364

11. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is

(a) $\frac{n(4n^2 - 1)c^2}{6}$ (b) $\frac{n(4n^2 + 1)c^2}{3}$

(c) $\frac{n(4n^2 - 1)c^2}{3}$ (d) $\frac{n(4n^2 + 1)c^2}{6}$

MORE THAN ONE OPTIONS CORRECT TYPE

12. The value(s) of 'a' for which the area of the triangle included between the axes and any tangent to the curve $x^a y = \lambda^a$ is constant, is/are

(a) $-\frac{1}{2}$ (b) -1 (c) $\frac{1}{2}$ (d) 1

13. Let $\vec{a}$, b and $\vec{c}$ be three non-coplanar vectors and $\vec{d}$ be a non-zero vector, which is perpendicular to $(\vec{a}+\vec{b}+\vec{c})$. Now $\vec{d} = (\vec{a}\times\vec{b})\sin x + (\vec{b}\times\vec{c})\cos y + 2(\vec{c}\times\vec{a})$. Then

(a) $\frac{\vec{d}\cdot(\vec{a}+\vec{c})}{[\vec{a}\ \vec{b}\ \vec{c}]} = 2$ (b) $\frac{\vec{d}\cdot(\vec{a}+\vec{c})}{[\vec{a}\ \vec{b}\ \vec{c}]} = -2$

(c) minimum value of $x^2 + y^2$ is $\pi^2/4$

(d) minimum value of $x^2 + y^2$ is $5\pi^2/4$

14. If $x_n = (\cos\theta)^{1/2^n} + (\sin\theta)^{1/2^n}$,

$y_n = (\cos\theta)^{1/2^n} - (\sin\theta)^{1/2^n}$ and

$I = \int_0^{\alpha}\left(y_n \prod_{r=1}^{n} x_r\right) d\theta$, then which of the following intervals belong to solution set of I ?

(a) $\left[-\sqrt{2}-1, \sqrt{2}-1\right]$ (b) $[-1, 0]$

(c) $\left[-\sqrt{2}, \sqrt{2}\right]$ (d) $\left[-\sqrt{2}, 0\right]$

15. Given that $x \in [0, 1]$ and $y \in [0, 1]$. Let A be the event of (x, y) satisfying $y^2 \le x$ and B be the event of (x, y) satisfying $x^2 \le y$. Then

(a) $P(A \cap B) = \frac{1}{3}$

(b) A, B are not exhaustive

(c) A, B are mutually exclusive

(d) A, B are not independent.

COMPREHENSION TYPE

Paragraph for Q. No. 16 and 17

Let S be a finite subset of rational numbers

$S = \{x_1, x_2, x_3, ..., x_n\}$ and $f(x) = x - \frac{1}{x}$. Define

$R_1 = \{f(x_1), f(x_2), f(x_3), ..., f(x_n)\}$ and $S_1 = R_1 \cap S$;

$R_2 = \{fof(x_1), fof(x_2), fof(x_3), ..., fo(f(x_n)\}$ and

$S_2 = R_2 \cap S$ and so on (S does not contain $-1, 0, 1$).

16. Which of the following is not correct?

(a) If $n(S) = 2^k$, then $n(R_1)$ may be 2^{k-1}

(b) If $n(S) = k$, then $R_k \neq \phi$.

(c) If $n(S) = 2^k$, then $n(R_1)$ can not be less than 2^{k-1}.

(d) If $n(S) = k$, then $S_{k+1} = \phi$.

17. Which of the the following is correct?

(a) It is possible to select S such that $R_1 \cap R_2 \cap R_3 ... \infty \neq \phi$.

(b) There exists S such that $S_1 \cap S_2 \cap S_3 .... \infty \neq \phi$.

(c) There exists $n \in N$ such that $S_n = S_{n+1} = S_{n+2} = ... \infty = \phi$.

(d) There exists $n \in N$ such that $R_n = R_{n+1} = R_{n+2} ... \infty = \phi$.

Paragraph for Q. No. 18 to 20

Consider the triangle with vertices $A(8, 0)$, $B(0, 6)$, $C(0, 0)$. Equilateral triangles BCD, CAE, ABF are constructed all externally on the sides of the triangle. Let G_1, G_2, G_3 be the centroids of the triangles BCD, CAE, ABF respectively.

18. If $F = (x, y)$, then $x - y =$

(a) 1 (b) $\sqrt{3}$

(c) $1+\sqrt{3}$ (d) $1-\sqrt{3}$

19. $G_2\ G_3 =$

(a) $\frac{2}{\sqrt{3}}\left(25+12\sqrt{3}\right)^{\frac{1}{2}}$ (b) $\frac{4}{\sqrt{3}}\left(25+12\sqrt{3}\right)^{\frac{1}{2}}$

(c) $\frac{2}{\sqrt{3}}\left(12+25\sqrt{3}\right)^{\frac{1}{2}}$ (d) $\frac{4}{\sqrt{3}}\left(12+25\sqrt{3}\right)^{\frac{1}{2}}$

20. The area of triangle $G_1\ G_2\ G_3$ is

(a) $12 + 25\sqrt{3}$ (b) $12 + \frac{25}{\sqrt{3}}$

(c) $25 + 12\sqrt{3}$ (d) $25 + 4\sqrt{3}$

Paragraph for Q. No. 21 and 22

Consider $f(x) = ke^x - x$ for all real x, where k is a real constant.

21. The line $y = x$ meets $y = ke^x$ for $k \le 0$ at

(a) no point

(b) one point

(c) two points

(d) more than two points

22. The positive value of k for which $ke^x - x = 0$ has only one root is

(a) $\frac{1}{e}$ (b) 1 (c) e (d) $\log_e 2$

MATRIX MATCH TYPE

23. Match the following.

Column - I		Column - II	
(P)	$\int_0^{\infty} e^{-4x} \sin 5x dx$	(1)	3
(Q)	$\int_2^8 \frac{[x^2]dx}{[x^2-20x+100]+[x^2]}$ (where [.] is greater integer function)	(2)	$\frac{5}{41}$
(R)	$\int_0^{3\pi/2} \lvert\sin x\rvert dx$	(3)	120
(S)	$\int_0^{\infty} x^5 e^{-x} dx$	(4)	60

	P	Q	R	S
(a)	2	1	4	3
(b)	2	1	1	3
(c)	1	2	3	4
(d)	1	4	2	3

24. If AB is a chord of the parabola $y^2 = 4x$ such that the normals at A and B intersect at the point $C(9, 6)$, then match the following:

Column-I		Column-II	
(P)	The length of AB is	(1)	20
(Q)	The area of ΔABC is	(2)	$\frac{4}{\sqrt{13}}$
(R)	The distance of the origin from the line AB is	(3)	$\sqrt{13}$
(S)	The area bounded by the coordinate axes and the line AB is	(4)	$\frac{4}{3}$

	P	Q	R	S
(a)	1	2	3	4
(b)	1	3	2	4
(c)	3	1	2	4
(d)	3	2	1	4

25. If $f(x, y, z) = \frac{x}{y-z} + \frac{y}{z-x} + \frac{z}{x-y}$, $(x < y < z \in R)$, then match the following:

Column-I		Column-II	
(P)	$f_x + f_y + f_z$ may be	(1)	zero
(Q)	$xf_x + yf_y + zf_z$ may be	(2)	positive
(R)	If $\sum f_x = \lambda\left\{\frac{(x-y)}{(y-z)(z-x)} + \frac{(y-z)}{(x-y)(z-x)} + \frac{z-x}{(x-y)(y-z)}\right\}$, then λ is	(3)	negative
(S)	$x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy}$ may be	(4)	real

	P	Q	R	S
(a)	1, 4	2, 3	3, 4	1, 2
(b)	3, 4	1, 4	2, 4	3, 4
(c)	1, 4	2, 3	2, 4	1, 4
(d)	3, 4	1, 4	3, 4	1, 4

NUMERICAL ANSWER TYPE

26. A box contains N coins of which m are fair and rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. If the probability that the coin drawn is fair is $f(N, m)$, then the value of $43\, f(20, 12)$ must be _____.

27. ABC is an isosceles triangle inscribed in a circle of radius r. If $AB = AC$ and h is the altitude from A to BC. If the triangle ABC has perimeter P and area Δ, then $\lim_{h\to 0} 512r\frac{\Delta}{P^3}$ is equal to _____.

28. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = \frac{4y^3 + 2yx^2}{3xy^2 + x^3}$, $y(1) = 1$. If for some $n \in N$, $y(2) \in [n-1, n)$, then n is equal to _____.

29. Two distinct numbers are chosen at random from the set $\{1, 2, \ldots\ldots, 3n\}$. The probability that $x^2 - y^2$ is divisible by 3 is $\frac{pn+q}{r(3n-1)}$, then the value of $p + q + r$ is _____.

30. The probability of at least one double-six being thrown in n throws with two ordinary dice is greater than 99 percent. Then the least numerical value of n is _____.

SOLUTIONS

1. (a): Let the point P be (x, y, z), then the vector $x\hat{i} + y\hat{j} + z\hat{k}$ will lie on the line.

Thus, $(x-1)\hat{i} + (y-1)\hat{j} + (z-1)\hat{k} = -\lambda\hat{i} + \lambda\hat{j} - \lambda\hat{k}$

$\Rightarrow x = 1 - \lambda$, $y = 1 + \lambda$ and $z = 1 - \lambda$

Now, point P is nearest to the origin

$\Rightarrow \quad D = (1-\lambda)^2 + (1+\lambda)^2 + (1-\lambda)^2$

or $\quad \dfrac{dD}{d\lambda} = -4(1-\lambda) + 2(1+\lambda) = 0 \Rightarrow \lambda = \dfrac{1}{3}$

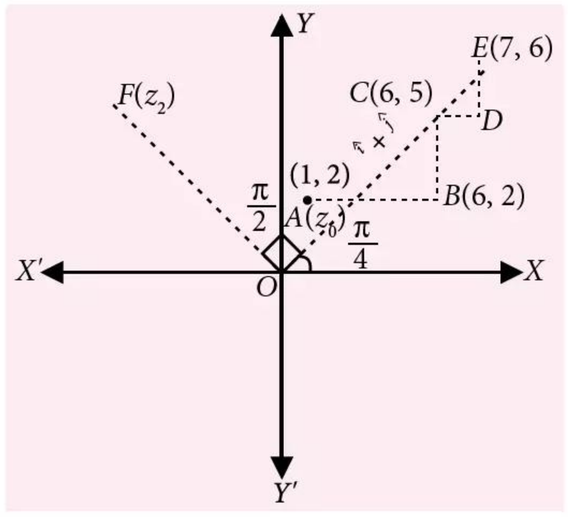
Hence, the point is $\left(\dfrac{2}{3}, \dfrac{4}{3}, \dfrac{2}{3}\right)$.

2. (d): It is given that CE is in the direction of vector $\hat{i} + \hat{j}$ which makes 45° with X-axis.

Also, $CE = \sqrt{2}$.

$\therefore \quad CD = DE = 1$ unit

So, the coordinates of E are (7, 6).



OE is rotated through an angle $\pi/2$ in anticlockwise direction to reach a point $F(z_2)$.

$z_2 = \overrightarrow{OE}\, e^{i\pi/2} = (7+6i)i = -6+7i$

3. (b): We have, $2x - y + z = 0$, $x - 2y + z = 0$, $tx - y + 2z = 0$

Since, the given system of equations has infinitely many solutions.

$\therefore \quad \Delta_x = \Delta_y = \Delta_z = \Delta = 0$

$$\Rightarrow \quad \begin{vmatrix} 2 & -1 & 1 \\ 1 & -2 & 1 \\ t & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \quad 2(-4+1) + 1\,(2-t) + 1\,(-1+2t) = 0$

$\Rightarrow \quad -6 + 2 - t - 1 + 2t = 0 \Rightarrow -5 + t = 0 \Rightarrow t = 5$

Now, $\displaystyle\int_0^{-2t} f(x)dx = \int_0^{-10} f(x)dx = -\int_{-10}^{0} f(x)\,dx$

$$= -\left[\int_{-10}^{-5} f(x)\,dx + \int_{-5}^{0} f(x)dx\right]$$

$$= -\left[\int_{-10}^{-5} f(x)\,dx + \int_{-10}^{-5} f(x+5)\,dx\right]$$

$$= -\int_{-10}^{-5} \left[f(x) + f(x+5)\right]dx$$

$$= -\int_{-10}^{-5} 2dx = -2\left[x\right]_{-10}^{-5} = -10 = -2t$$

4. (c): Let A denote the event that target is hit when x shells are fired at point I. Let P_1 and P_2 denote the event that the target is at point I and II, respectively.

We have, $P(P_1) = 8/9$, $P(P_2) = 1/9$, $P(A|P_1) = 1 - (1/2)^x$, $P(A|P_2) = 1 - (1/2)^{55-x}$

By total probability theorem,

$P(A) = P(P_1)\,P(A|P) + P(P_2)\,P(A|P_2)$

$$= \frac{1}{9}\left(8 - 8\left(\frac{1}{2}\right)^x + 1 - \left(\frac{1}{2}\right)^{55-x}\right)$$

$$= \frac{1}{9}\left(9 - 8\left(\frac{1}{2}\right)^x - \left(\frac{1}{2}\right)^{55-x}\right)$$

Now, $\dfrac{dP(A)}{dx} = \dfrac{1}{9}\left(-8\left(\dfrac{1}{2}\right)^x \ln\left(\dfrac{1}{2}\right) + \left(\dfrac{1}{2}\right)^{55-x} \ln\left(\dfrac{1}{2}\right)\right)$

$$= \frac{1}{9}\ln\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{55-x}\left(1 - \left(\frac{1}{2}\right)^{2x-58}\right) > 0 \text{ if } x < 29 \text{ and } < 0 \text{ if } x > 29$$

5. (a): $\dfrac{V_n - V_{n-1}}{U_{n-1}}$

$$= \frac{\cos n\theta \cdot \sec^n\theta - \cos\{(n-1)\theta\} \cdot \sec^{n-1}\theta}{\sin\{(n-1)\theta\} \cdot \sec^{n-1}\theta}$$

$$= \frac{\sec^n\theta[\cos\{(n-1)\theta + \theta\} - \cos\{(n-1)\theta\} \cdot \cos\theta]}{\sec^n\theta \cdot \sin\{(n-1)\theta\} \cdot \cos\theta}$$

$$= \frac{\begin{array}{r}\cos\{(n-1)\theta\} \cdot \cos\theta - \sin\{(n-1)\theta\} \cdot \sin\theta \\ -\cos\{(n-1)\theta\}\cos\theta\end{array}}{\sin\{(n-1)\theta\}\cos\theta}$$

$$= \frac{-\sin\{(n-1)\theta\}\sin\theta}{\sin\{(n-1)\theta\}\cos\theta} = -\tan\theta$$

$\Rightarrow \quad V_n - V_{n-1} = -U_{n-1} \cdot \tan\theta$

$\Rightarrow \quad V_n - V_{n-1} + U_{n-1} \cdot \tan\theta = 0$

6. (d): Let $A = \lim\limits_{n\to\infty} \dfrac{1}{n}\left[(n+1)(n+2)(n+3)\ldots(n+n)\right]^{1/n}$

$$= \lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)\left(\frac{n+2}{n}\right)\left(\frac{n+3}{n}\right)\ldots\left(\frac{n+n}{n}\right)\right]^{1/n}$$

$$\Rightarrow \quad A = \lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\ldots\left(1+\frac{n}{n}\right)\right]^{1/n}$$

$$\Rightarrow \quad \log A = \lim_{n-\infty} \frac{1}{n}\prod_{r=1}^{n} \log\left(1+\frac{r}{n}\right)$$

$$\Rightarrow \quad \log A = \int_0^1 \log(1+x)dx = \left[x\log(1+x)\right]_0^1 - \int_0^1 \frac{x}{x+1}dx$$

$$\Rightarrow \quad \log A = \log 2 - \int_0^1 \left(1 - \frac{1}{x+1}\right)dx$$

$\Rightarrow \log A = \log 2 - 1 + \log 2 = \log\left(\frac{4}{e}\right)$

$\Rightarrow A = 4/e$

7. (c) : If exactly r elements have self-image, then number of functions = (Number of ways of selecting r elements) × (De-rangement of remaining elements)

$= {}^nC_r \times (n-r)!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \ldots\right)$

$\therefore$ Required number of functions

$= {}^6C_3 \times 3! \times \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right) + {}^6C_4 \times 2!\left(1 - \frac{1}{1!} + \frac{1}{2!}\right)$

$+ {}^6C_5 \times 1! \times \left(1 - \frac{1}{1!}\right) + {}^6C_6 = 40 + 15 + 0 + 1 = 56$

8. (c) : We have, $f(x) = \alpha(x)\,\beta(x)\,\gamma(x)$, for all real x

$\Rightarrow f'(x) = \alpha'(x)\beta(x)\gamma(x) + \alpha(x)\beta'(x)\gamma(x) + \alpha(x)\,\beta(x)\,\gamma'(x)$

$\Rightarrow f'(2) = \alpha'(2)\beta(2)\gamma(2) + \alpha(2)\beta'(2)\gamma(2) + \alpha(2)\beta(2)\gamma'(2)$

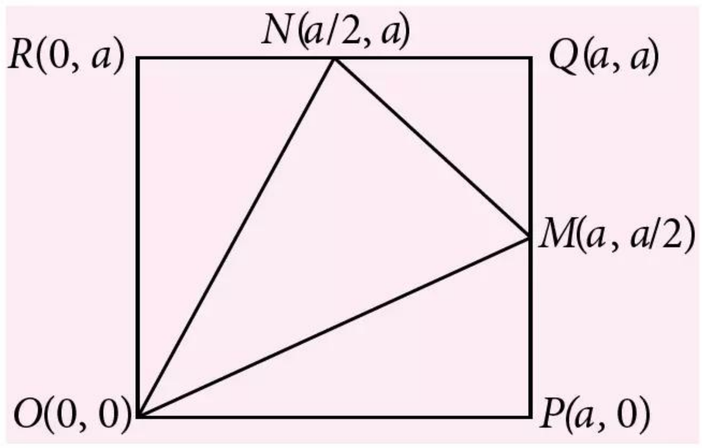
$\Rightarrow 18\,f(2) = 3\alpha(2)\,\beta\,(2)\,\gamma(2) - 4\alpha(2)\,\beta(2)\gamma(2) + k\,\alpha(2)\,\beta(2)\,\gamma(2)$

[$\because f'(2) = 18\,f(2)$, $\alpha'(2) = 3\alpha(2)$, $\beta'(2) = -4\beta(2)$ and $\gamma'(2) = k\,\gamma\,(2)$]

$\Rightarrow 18f(2) = (-1 + k)\alpha(2)\beta(2)\gamma(2) = (k-1)f(2)$ [$\because f(2) = \alpha(2)\beta(2)\gamma(2)$]

$\Rightarrow k - 1 = 18 \Rightarrow k = 19$

9. (c) : Let the coordinates of vertices O, P, Q, R as $(0, 0)$, $(a, 0)$, (a, a), $(0, a)$ respectively then the coordinates of M as $(a, a/2)$ and of N as $(a/2, a)$.



$\therefore$ Area of $\Delta OMN = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & a/2 & 1 \\ a/2 & a & 1 \end{vmatrix} = \frac{3a^2}{8}$

Area of square $= a^2$

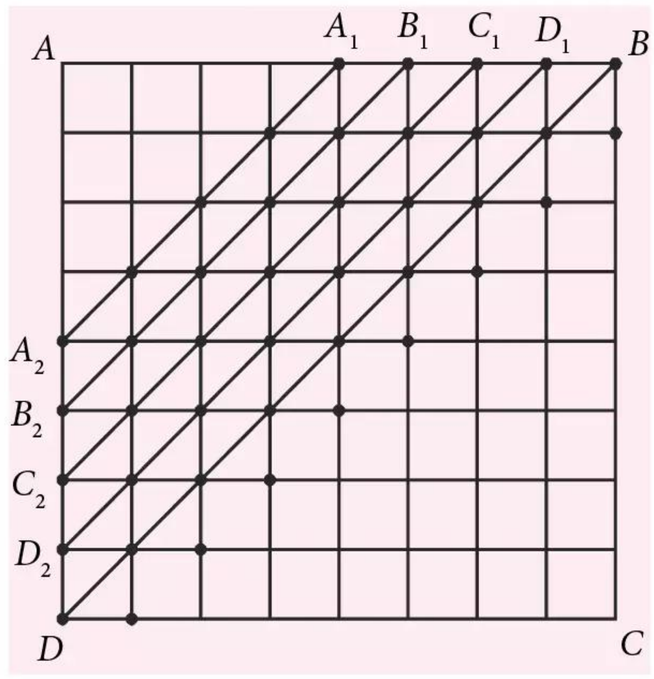
$\therefore$ The required ratio is 8 : 3.

10. (d): Total number of squares in a chess board $= 8 \times 8 = 64$

$\therefore$ Number of selecting 4 squares $n(S) = C(64, 4)$

The diagonal BD or AC divide the chess board in two equal parts if we start to choose (select) four squares from diagonal A_1A_2, B_1B_2, C_1C_2, …, BD, then total selected squares are given by

$${}^4C_4 + {}^5C_4 + {}^6C_4 + {}^7C_4 + {}^8C_4 + {}^7C_4 + {}^6C_4 + {}^5C_4 + {}^4C_4$$
$$= 2({}^4C_4 + {}^5C_4 + {}^6C_4 + {}^7C_4) + {}^8C_4$$
$$= 2(1 + 5 + 15 + 35) + 70 = 112 + 70 = 182$$



Similarly, same number of squares can be selected from the diagonal AC.

$\therefore$ Total number of favourable cases $= 2 \times 182 = 364$

Now, required probability $= \frac{\text{Favourable cases}}{\text{Total cases}} = \frac{364}{C(64, 4)}$

According to problem, we have $\frac{m}{C(64, 4)} = \frac{364}{C(64, 4)}$

$\Rightarrow m = 364$

11. (c) : $S_n = cn^2$, $t_n = S_n - S_{n-1} = c(n^2 - (n-1)^2)$

$\therefore t_n = c(2n - 1)$

Sum $= t_1^{\,2} + t_2^{\,2} + \ldots\ldots + t_n^{\,2}$

$$= c^2 \sum_{r=1}^{n} (2r-1)^2 = c^2[4\Sigma r^2 - 4\Sigma r + \Sigma 1]$$

$$= c^2\left[\frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n\right]$$

$$= \frac{c^2 n}{6}[8n^2 + 12n + 4 - 12n - 12 + 6]$$

$$= \frac{c^2 n}{3}(4n^2 - 1)$$

12. (b, d) : Given curve, $x^a\,y = \lambda^a$ ...(i)

$(\lambda, 1)$ is a point on the given curve.

Now, differentiating (i) w.r.t. x, we get

$$ax^{a-1}\,y + x^a\,\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-ax^{a-1}y}{x^a} = -\frac{ay}{x}$$

$$\left[\frac{dy}{dx}\right]_{(\lambda, 1)} = -\frac{a}{\lambda}$$

Equation of tangent at $(\lambda, 1)$ is $y - 1 = -\frac{a}{\lambda}(x - \lambda)$

Now, $x = 0 \Rightarrow y = 1 + a$,

$$y = 0 \Rightarrow \quad x = \frac{\lambda}{a} + \lambda = \frac{\lambda(1+a)}{a}$$

$$\therefore \quad \text{Area, } A = \frac{1}{2} \times (1+a)\frac{(a+1)\lambda}{a}$$

$$\text{Now, } \frac{dA}{da} = \frac{1}{2}\lambda\left[\frac{a \cdot 2(1+a) - (1+a)^2}{a^2}\right] = 0$$

$\Rightarrow (2a - 1 - a)(1 + a) = 0$

$\Rightarrow (a - 1)(a + 1) = 0 \Rightarrow a = 1, a = -1$

13. (b, d) : We have, $\vec{d} \cdot \vec{a} = [\vec{a}\ \vec{b}\ \vec{c}]\cos y = -\vec{d} \cdot (\vec{b} + \vec{c})$

$$\Rightarrow \quad \cos y = -\frac{\vec{d} \cdot (\vec{b} + \vec{c})}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

Similarly, $\sin x = \dfrac{\vec{d} \cdot (\vec{a} + \vec{b})}{[\vec{a}\ \vec{b}\ \vec{c}]}$ and $\dfrac{\vec{d} \cdot (\vec{a} + \vec{c})}{[\vec{a}\ \vec{b}\ \vec{c}]} = -2$

$\therefore \quad \sin x + \cos y + 2 = 0 \Rightarrow \sin x + \cos y = -2$

$\Rightarrow \sin x = -1, \cos y = -1$

Since, we want the minimum value of $x^2 + y^2$,

$x = -\pi/2, y = \pi$

Therefore, the minimum value of $x^2 + y^2$ is $5\pi^2/4$.

14. (a, b, d) : $x_n y_n = \{(\cos\theta)^{1/2^n}\}^2 - \{(\sin\theta)^{1/2^n}\}^2$

$= (\cos\theta)^{1/2^{n-1}} - (\sin\theta)^{1/2^{n-1}} = y_{n-1}$

Similarly, $x_{n-1} \cdot y_{n-1} = y_{n-2}$ ....

$$\therefore I = \int_0^\alpha y_n \left(x_1 \cdot x_2 \cdot x_3 \ \ldots\ldots\ x_n\right) d\theta$$

$$= \int_0^\alpha x_1 \cdot x_2 \cdot x_3 \ldots\ldots \left(x_n \cdot y_n\right) d\theta$$

$$= \int_0^\alpha x_1 \cdot x_2 \cdot x_3 \ \ldots\ldots x_{n-1}\left(y_{n-1}\right) d\theta$$

$$= \int_0^\alpha x_1 \cdot x_2 \cdot x_3 \ \ldots\ldots \left(x_{n-2} \cdot y_{n-2}\right) d\theta$$

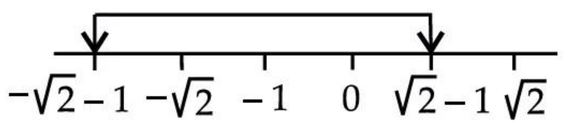
$$= \ldots\ldots\ldots = \int_0^\theta x_1 \cdot y_1 \, d\theta$$

$$= \int_0^\alpha \left\{(\cos\theta)^{1/2} + (\sin\theta)^{1/2}\right\}\left\{(\cos\theta)^{1/2} - (\sin\theta)^{1/2}\right\} d\theta$$

$$= \int_0^\alpha \left(\cos\theta - \sin\theta\right) d\theta = \sin\alpha + \cos\alpha - 1$$

$$= \sqrt{2}\sin\left(\alpha + \frac{\pi}{4}\right) - 1$$

$$\therefore I \in \left[-\sqrt{2} - 1, \sqrt{2} - 1\right]$$

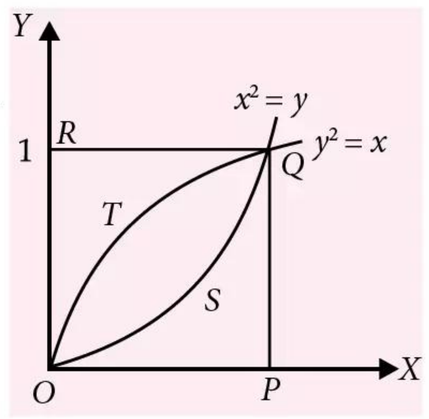


15. (a, b, d) : A = the event of (x, y) belonging to the area $OTQPO$

B = the event of (x, y) belonging to the area $OSQRO$

$$P(A) = \frac{\text{ar}\,(OTQPO)}{\text{ar}\,(OPQRO)} = \frac{\int_0^1 \sqrt{x}\,dx}{1 \times 1} = \left[\frac{2}{3}x^{3/2}\right]_0^1 = \frac{2}{3}$$

$$P(B) = \frac{\text{ar}(OSQRO)}{\text{ar}(OPQRO)} = \frac{\int_0^1 \sqrt{y}\,dy}{1 \times 1} = \frac{2}{3}$$



$$P(A \cap B) = \frac{\text{ar}\,(OTQS)}{\text{ar}\,(OPQRO)}$$

$$= \frac{\int_0^1 \sqrt{x}\,dx - \int_0^1 x^2\,dx}{1 \times 1} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$P(A) + P(B) = \frac{2}{3} + \frac{2}{3} \neq 1$$

So, A and B are not exhaustive

$$P(A) \cdot P(B) = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9} \neq P(A \cap B)$$

So, A and B are not independent.

$$P(A \cup B) = 1, P(A) + P(B) = \frac{2}{3} + \frac{2}{3} \neq P(A \cup B)$$

So, A and B are not mutually exclusive.

16. (d): $f(x_1) = f(x_2) = f(x_3)$ is not possible for three distinct x_1, x_2, x_3.

17. (c) : For $x_i = \dfrac{p}{q}$ (in its lowest form) define $g(x_i) = |p| + |q|$

Now, $g(f(x)) = |p^2 - q^2| + |pq|$

So, $g(f(x)) - g(x) = |(p^2 - q^2)| + |pq| - |p| - |q|$

$\leq 3 + |p|\,|q| - |p| - |q| = (|p| - 1)\,(|q| - 1) + 2 \leq 2$

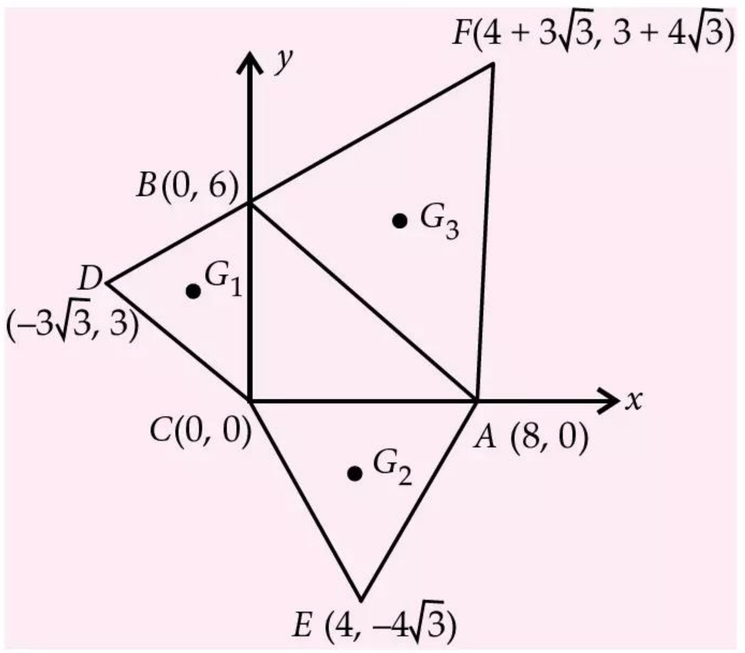
Hence, for sufficiently large n, $R_n \cap S$ will be ϕ.

18. (d): Here, $D = (-3\sqrt{3}, 3)$, $E = (4, -4\sqrt{3})$,

$F = (4 + 3\sqrt{3}, 3 + 4\sqrt{3})$,

$G_1 = (-\sqrt{3}, 3)$, $G_2 = \left(4, \dfrac{-4}{\sqrt{3}}\right)$, $G_3 = \left(4 + \sqrt{3}, 3 + \frac{4}{\sqrt{3}}\right)$

$x - y = 4 + 3\sqrt{3} - (3 + 4\sqrt{3}) = 1 - \sqrt{3}$



19. (a) : $G_2 = \left(4, \dfrac{-4}{\sqrt{3}}\right)$, $G_3 = \left(4 + \sqrt{3}, 3 + \dfrac{4}{\sqrt{3}}\right)$

$$(G_2\,G_3)^2 = 3 + \left(3 + \frac{8}{\sqrt{3}}\right)^2 = 12 + \frac{64}{3} + 16\sqrt{3}$$

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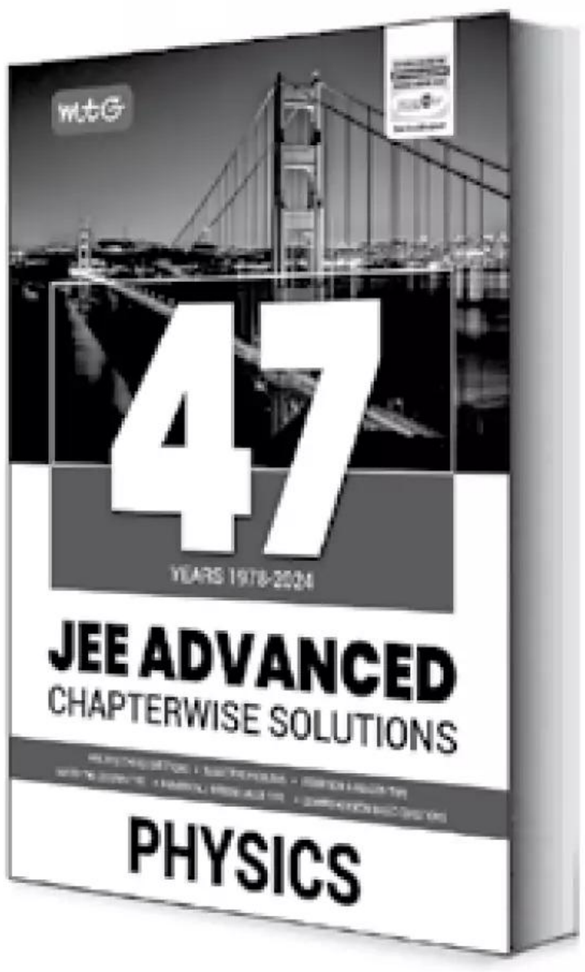

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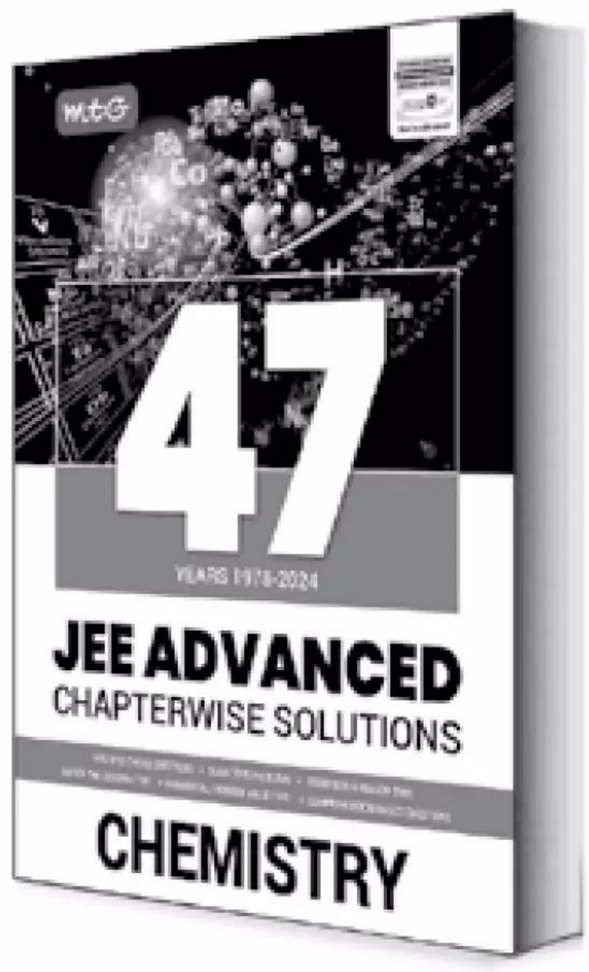

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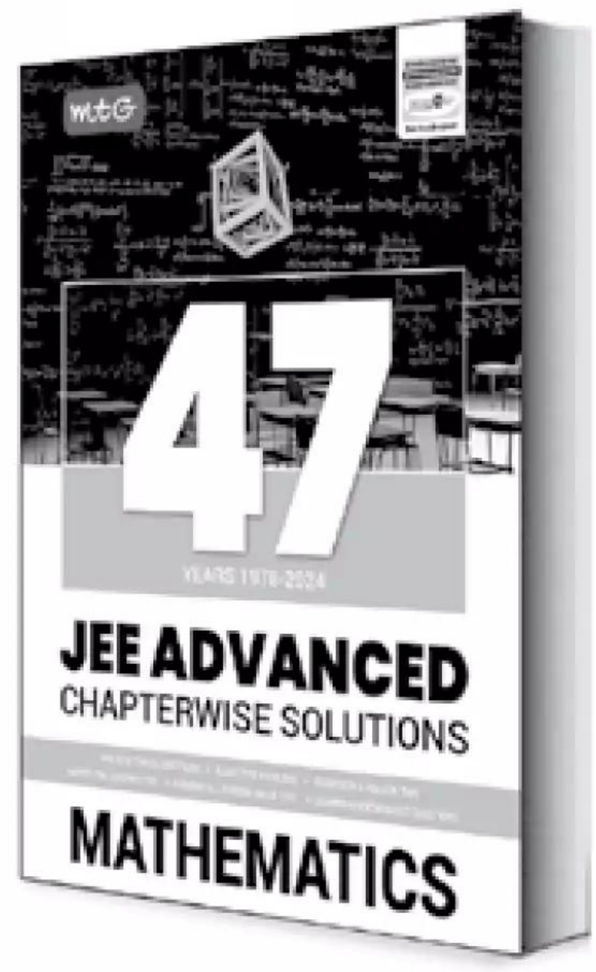

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$$= \frac{100}{3} + 16\sqrt{3} \Rightarrow G_2G_3 = \frac{2}{\sqrt{3}}\left(25 + 12\sqrt{3}\right)^{\frac{1}{2}}$$

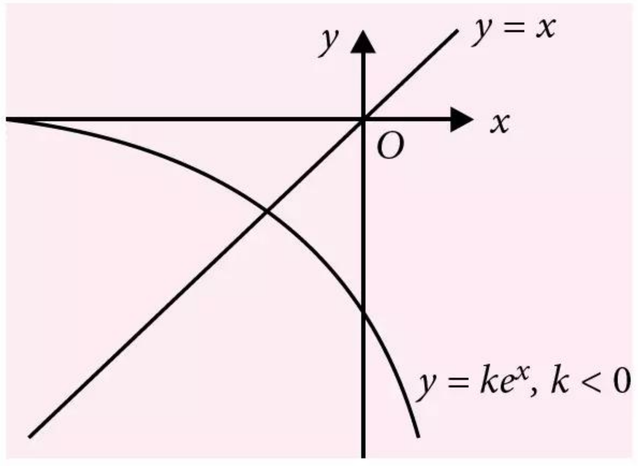
20. (b): $G_1G_2 = G_2G_3 = G_3G_1 = \frac{2}{\sqrt{3}}\left(25 + 12\sqrt{3}\right)^{\frac{1}{2}}$

$$\text{Area} = \frac{\sqrt{3}}{4}(G_1G_2)^2 = 12 + \frac{25}{\sqrt{3}}$$

21. (b): We have, $y = k \cdot e^x$

For $k = 0$, line $y = x$ meets $y = 0$, *i.e.*, the x-axis, only at one point.

For $k < 0$, $y = ke^x$ meets $y = x$ only once as shown in the figure.



22. (a): Let $f(x) = ke^x - x$

Now, for $f(x) = 0$ and to have only one root means the line $y = x$ must be tangential to the curve $y = ke^x$.

Let it be so at (x_1, y_1). Then

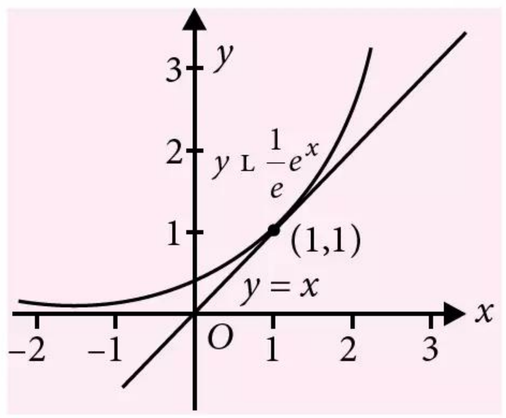
$$\left(\frac{dy}{dx}\right)_{x_1} = ke^{x_1} = 1 \quad (\because \quad y = x \text{ touches curve at } (x_1, y_1))$$

$$\Rightarrow \quad e^{x_1} = \frac{1}{k}$$

Also, $y_1 = ke^{x_1}$ and $y_1 = x_1$

$\therefore \quad x_1 = 1$

$\Rightarrow 1 = ke \Rightarrow k = 1/e$



23. (b): **(P)** $\int_0^\infty e^{-4x} \sin 5x \, dx$

$$= \left[\frac{e^{-4x}}{5^2 + 4^2}[-4\sin 5x - 5\cos 5x\right]_0^\infty = \frac{5}{41}$$

(Q) $I = \int_2^8 \frac{[x^2]dx}{\left[(x-10)^2\right] + \left[x^2\right]}$

$$= \int_2^8 \frac{\left[(10-x)^2\right]dx}{\left[(10-x-10)^2\right] + \left[(10-x)^2\right]}$$

$$\therefore \quad 2I = \int_2^8 \frac{[x^2] + \left[(10-x)^2\right]}{\left[(x-10)^2\right] + \left[x^2\right]} dx = \int_2^8 dx \quad \Rightarrow \quad I = 3$$

(R) $\int_0^{3\pi/2} |\sin x| dx = \int_0^{\pi} \sin x \, dx - \int_{\pi}^{3\pi/2} \sin x \, dx = 3$

(S) $\int_0^\infty x^5 e^{-x} dx$

$$= -e^{-x}(x^5 + 5x^4 + 20x^3 + 60x^2 + 120x + 120)\Big|_0^\infty = 120$$

24. (c): **(P)** If the parameter t_1, t_2, t_3 correspond to the points A, B, C respectively, then $t_3 = 3$, $t_1 + t_2 = -3$, $t_1t_2 = 2$

$\Rightarrow \quad t_1 = -1, t_2 = -2$

$AB^2 = (t_1 - t_2)^2[(t_1 + t_2)^2 + 4] = 13 \quad \Rightarrow AB = \sqrt{13}$

(Q) Equation of AB is $2x - (t_1 + t_2)y + 2t_1t_2 = 0$

$\Rightarrow \quad 2x + 3y + 4 = 0$

The distance of $C(9, 6)$ from this line is $\frac{40}{\sqrt{13}}$

$\therefore$ Area of $\Delta ABC = \frac{\sqrt{13}}{2} \cdot \frac{40}{\sqrt{13}} = 20$

(R) Distance of $(0, 0)$ from the line

$2x + 3y + 4 = 0$ is $\frac{4}{\sqrt{13}}$

(S) The line $2x + 3y + 4 = 0$ makes intercepts of 2, 4/3 on the axes.

$\therefore$ The area is 4/3.

25. (d): $f = \frac{x}{y-z} + \frac{y}{z-x} + \frac{z}{x-y}$, clearly $x \neq y \neq z$

$$f_x = \frac{1}{y-z} + \frac{y}{(z-x)^2} - \frac{z}{(x-y)^2}$$

$$\Sigma f_x = \Sigma \frac{1}{y-z} + \Sigma\left[\frac{y}{(z-x)^2} - \frac{z}{(x-y)^2}\right]$$

$$= \Sigma \frac{1}{y-z} + 0 = \Sigma \frac{1}{y-z}$$

$$= \frac{(xy + yz + zx) - (x^2 + y^2 + z^2)}{(x-y)(y-z)(z-x)}$$

$$= \frac{-\frac{1}{2}\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right]}{(x-y)(y-z)(z-x)}, \text{ Negative}$$

$\therefore \quad \lambda = -\frac{1}{2}$. Also by Euler's theorem for homogeneous functions of degree '0' $\Sigma x f_x = 0$

Also, $x^2 f_{xx} + 2xyf_{xy} + y^2 f_{yy} = 0$

26. (27): Let E_1, E_2 and A denote the following events:

E_1 : Coin selected is fair.

E_2 : Coin selected is biased.

A : The first toss results in a head and the second toss results in a tail.

$$P(E_1)=\frac{m}{N};\ P(E_2)=\frac{N-m}{N}$$

$$\therefore\ P\left(\frac{A}{E_1}\right)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}\ \text{and}\ P\left(\frac{A}{E_2}\right)=\frac{2}{3}\times\frac{1}{3}=\frac{2}{9}$$

By Bayes' theorem,

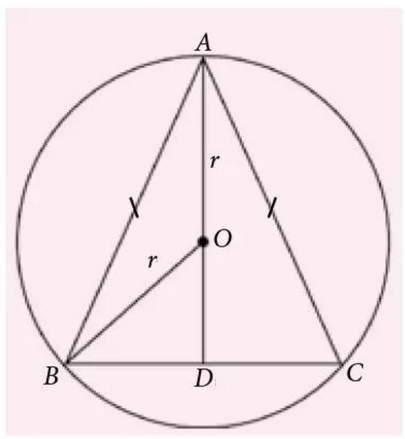
$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)}$$

$$=\frac{\frac{m}{N}\times\frac{1}{4}}{\frac{m}{N}\times\frac{1}{4}+\left(\frac{N-m}{N}\right)\times\frac{2}{9}}=\frac{9m}{8N+m}=f\ (N,m)\ \text{(given)}$$

$$\therefore\ \ f\ (20,\ 12)=\frac{9\times12}{8\times20+12}=\frac{108}{172}=\frac{27}{43}$$

$$\Rightarrow\ 43\ f\ (20,\ 12)=27$$

27. (4): We have, $BC = 2BD$, $AD = h$ and $OD = h - r$



$$\therefore\ \ BC=2\sqrt{r^2-(h-r)^2}$$

$$=2\sqrt{2hr-h^2}$$

$$\Rightarrow\ \ AB=\sqrt{2hr-h^2+h^2}=\sqrt{2hr}$$

Now, Perimeter $P = 2AB + BC$

$$=2[\sqrt{2hr-h^2}+\sqrt{2hr}]$$

Also, the area of ΔABC is $\Delta = BD\times AD=h\sqrt{2hr-h^2}$

$$\therefore\ \ \frac{\Delta}{P^3}=\frac{h\sqrt{2hr-h^2}}{8(\sqrt{2hr-h^2}+\sqrt{2hr})^3}=\frac{\sqrt{2r-h}}{8(\sqrt{2r-h}+\sqrt{2r})^3}$$

$$\Rightarrow\ \lim_{h\to0}512r\frac{\Delta}{P^3}=512r\frac{\sqrt{2r}}{8(2\sqrt{2r})^3}=4$$

28. (3): The given differential equation is

$$\frac{dy}{dx}=\frac{4y^3+2yx^2}{3xy^2+x^3}$$

It can be represented as $\frac{dy}{dx}=\frac{4+2\left(\frac{x}{y}\right)^2}{3\left(\frac{x}{y}\right)+\left(\frac{x}{y}\right)^3}$

Let $v=\frac{y}{x}\ \Rightarrow\ \frac{dy}{dx}=v+x\frac{dv}{dx}=\frac{4+2\left(\frac{1}{v}\right)^2}{\frac{3}{v}+\left(\frac{1}{v}\right)^3}$

$$\Rightarrow\ x\frac{dv}{dx}=\frac{4v^3+2v-3v^3-v}{3v^2+1}=\frac{v^3+v}{3v^2+1}$$

$$\Rightarrow\ \int\left(\frac{3v^2+1}{v^3+v}\right)dv=\int\frac{dx}{x}$$

Let $v^3+v=t\Rightarrow(3v^2+1)dv=dt\ \ \Rightarrow\ \int\frac{dt}{t}=\ln x+\ln c$

$\Rightarrow\ \ln|t|=\ln(x\cdot c)\Rightarrow|v^3+v|=x\cdot c$

$$\Rightarrow\ \left(\frac{y}{x}\right)^3+\left(\frac{y}{x}\right)=x\cdot c\ \Rightarrow\ \frac{y^3}{x^3}+\frac{y}{x}=x\cdot c$$

$$\Rightarrow\ y^3+yx^2=x^4\cdot c$$

As $y(1) = 1\ \Rightarrow 2 = c$

$$\therefore\ \ y^3+yx^2=2x^4$$

As $y(2)\in[n-1,\ n)\Rightarrow\ y^3+4y=32$

Let $f(y) = y^3 + 4y - 32$; $f(3) > 0$ and $f(2) < 0$

$\therefore\ \ f(2)\cdot f(3)<0$

$\therefore$ One root lies in $[2, 3)\ \ \therefore\ n = 3$

29. (5): $n(S) = {}^{3n}C_2$

Let $A_0 = \{3, 6, 9, \ldots 3n\}$, $A_1 = \{1, 4, 7, \ldots, 3n - 2\}$,

$A_2 = \{2, 5, 8, 11, \ldots, 3n - 1\}$

$(x^2 - y^2)$ is divisible by 3, if both x, y should come from A_0 or A_1 or A_2 or one is from A_1 and other from A_2.

$$n(E)=3\ {}^nC_2+{}^nC_1\times{}^nC_1=\frac{n}{2}(5n-3)$$

$$\therefore P(E)=\frac{5n-3}{3(3n-1)}=\frac{pn+q}{r(3n-1)}\ \Rightarrow\ p+q+r=5-3+3=5$$

30. (164): The probability of getting a double-six in one throw with two dice $=\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$

$$\therefore\ \ p=\frac{1}{36},\ q=1-p=1-\frac{1}{36}=\frac{35}{36}$$

Now, $(p+q)^n=p^n+{}^nC_1\ p^{n-1}q+{}^nC_2\ p^{n-2}\ q^2+\ldots+{}^nC_{n-1}pq^{n-1}+q^n$

The probability of getting atleast one double-six in n throws with two dice

$$=(p+q)^n-q^n=1-q^n=1-\left(\frac{35}{36}\right)^n$$

$$\therefore\ 1-\left(\frac{35}{36}\right)^n>0.99$$

$$\Rightarrow\ \left(\frac{35}{36}\right)^n<0.01\Rightarrow n[\log_{10}35-\log_{10}36]<\log_{10}0.01$$

$$\Rightarrow\ n[1.5441-1.5563]<-2.0000\Rightarrow-0.0122n<-2$$

$$\Rightarrow\ 0.0122n>2\ \Rightarrow\ n>\frac{2}{0.0122}\Rightarrow n>163.9$$

$\therefore$ The least value of n is 164.

JEE 2025 PRACTICE PAPER ADVANCED

Exam on 18th May

PAPER-I

SECTION 1 (MAXIMUM MARKS : 12)

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme :*

Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : –1 In all other cases.

1. A coin is tossed $2n$ times. The chance that the number of times one gets head is not equal to the number of times one gets tail is

(a) $\frac{(2n)!}{(n!)^2}\cdot\left(\frac{1}{2}\right)^{2n}$ (b) $1-\frac{(2n)!}{(n!)^2}$

(c) $1-\frac{(2n)!}{(n!)^2}\cdot\frac{1}{4^n}$ (d) none of these

2. Let A and B be two matrices such that $A=[a_{ij}]_{2\times 3}$

$$=\begin{bmatrix} x & 1 & 2 \\ 3x & -1 & \dfrac{1}{4x^2+1} \end{bmatrix} \text{ and } B=[b_{ij}]_{3\times 2}=\begin{bmatrix} \dfrac{1}{x^2} & \dfrac{1}{x} \\ 2x & 2 \\ 3 & x \end{bmatrix}$$

where $x>0$. If matrix C is defined as $C=[c_{ij}]_{2\times 2}$, where $c_{ij}=\sum_{r=1}^{3} a_{ir}b_{rj}\ \forall 1\le i,j\le 2$, then the minimum value of $\Delta(x)=\sum_{1\le i\le j\le 2} c_{ij}$ is

(a) 57/4 (b) 57/2

(c) 55/4 (d) 55/2

3. An urn contains 10 balls coloured either black or red. When selecting two balls from the urn at random, the probability that a ball of each colour is selected is 8/15. Assuming that the urn contains more black balls than red balls, the probability that atleast one black ball is selected, when selecting two balls, is

(a) 18/45 (b) 30/45

(c) 39/45 (d) 41/45

4. The value of $\lim_{n\to\infty}\left(\frac{n!}{(mn)^n}\right)^{1/n}$ is

(a) em (b) e/m

(c) $1/em$ (d) none of these

SECTION 2 (MAXIMUM MARKS : 12)

- *This section contains THREE (03) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

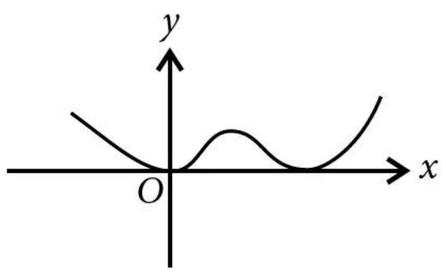
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

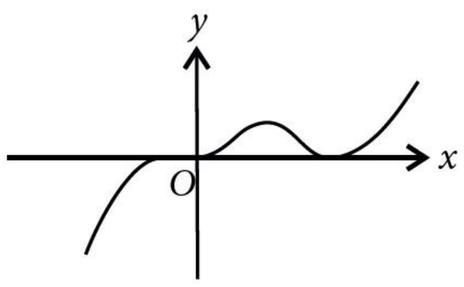
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : –2 In all other cases.

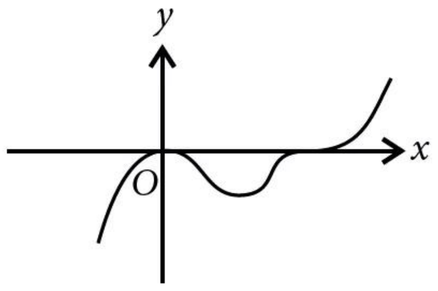
5. Let $f(x) = x^m(x-1)^n$, $m, n \in I$ and $(m, n > 1)$, then possible graph of $f(x)$ in various cases of m and n is/are

(a) 

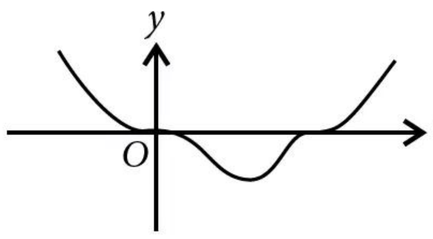
m even
n even

(b) 

m odd
n even

(c) 

m even
n odd

(d) 

m odd
n odd

6. Let x_1, x_2, x_3, x_4 be four non-zero numbers satisfying the equation $\tan^{-1}\frac{a}{x} + \tan^{-1}\frac{b}{x} + \tan^{-1}\frac{c}{x} + \tan^{-1}\frac{d}{x} = \frac{\pi}{2}$. Then

(a) $x_1 + x_2 + x_3 + x_4 = a + b + c + d$

(b) $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} \neq 0$

(c) $x_1x_2x_3x_4 = abcd$

(d) $(x_2 + x_3 + x_4)(x_3 + x_4 + x_1)(x_4 + x_1 + x_2)(x_1 + x_2 + x_3) = abcd$

7. A straight line through $P(\lambda, 2)$, $(\lambda \neq 0)$ meets the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at A and D meets the axes at B and C such that PA, PB, PC, PD are in G.P., then $\lambda \in$

(a) $[7, \infty)$ (b) $(-12, -8)$

(c) $(-5, 0)$ (d) $(10, \infty)$

SECTION 3 (MAXIMUM MARKS : 24)

- *This section contains SIX (06) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*
- *Answer to each question will be evaluated according to the following marking scheme:*

  Full Marks : +4 If ONLY the correct integer is entered;

  Zero Marks : 0 In all other cases.

8. If $|z| = r$ and z_1, z_2 are the roots of the equation $z^2 + |z|z + \bar{z}z = 0$, then $|z_1 - z_2|^2 = \lambda r^2$, where λ is _____.

9. If α is a complex seventh root of unity, then the equation whose roots are $\alpha + \alpha^2 + \alpha^4$ and $\alpha^3 + \alpha^5 + \alpha^6$ is $x^2 + x + c = 0$, where c is _____.

10. If $N = \sum_{1 \le r < s \le 20} (r-s)^2$, where $r, s \in \{1, 2, 3, ..., 20\}$, then the number of prime factors of N is _____.

11. If $\frac{N}{1!18!} = \frac{1}{2!17!} + \frac{1}{3!16!} + \frac{1}{4!15!} + \frac{1}{5!14!} + \frac{1}{6!13!} + \frac{1}{7!12!} + \frac{1}{8!11!} + \frac{1}{9!10!}$,

then the greatest integer less than $\frac{N}{10^4}$ is _____.

12. AB is a chord of the circle $x^2 + y^2 = 25$. The tangents at A and B meet at C. If $M(2, 3)$ is the midpoint of AB, then the area of the kite $OACB$ is $50\sqrt{\frac{k}{13}}$, then k is _____.

13. There are four balls of different colours and four boxes of colours, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own colour is _____.

SECTION 4 (MAXIMUM MARKS : 12)

- *This section contains FOUR (04) Matching List Sets.*
- *Each set has ONE Multiple Choice Question.*
- *Each set has TWO lists: List-I and List-II.*
- *List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).*
- *FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.*
- *Answer to each question will be evaluated according to the following marking scheme:*

  Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : –1 In all other cases.

14. In the triangle ABC, $C = \frac{\pi}{2}$, $r_1 = 11$ and $r_2 = 90$. **List-II** gives the number of divisors of the numbers in **List-I**.

List-I		List-II	
(P)	Δ	(1)	10
(Q)	r_3	(2)	8
(R)	a	(3)	12
(S)	b	(4)	24
		(5)	6

The correct option is

(a) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (5), (S) $\rightarrow$ (5)
(b) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (1), (R) $\rightarrow$ (2), (S) $\rightarrow$ (3)
(c) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (4), (S) $\rightarrow$ (1)
(d) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (1), (S) $\rightarrow$ (5)

15. Consider the ellipse $x^2 + 2y^2 = 2$. Let L be the end of the latus rectum in the first quadrant. The tangent at L to the ellipse meets the right side directrix at T. The normal at L meets the major axis at N and the left side directrix at M.

List-I		List-II	
(P)	The area of ΔLNT is	(1)	$\sqrt{2}$
(Q)	The circumradius of ΔLNT is	(2)	$\frac{1}{\sqrt{2}}$
(R)	The ratio $LN : LT$ is	(3)	3/4
(S)	The ratio $LN : NM$ is	(4)	$\frac{3}{4\sqrt{2}}$
		(5)	1/5

The correct option is

(a) (P) $\rightarrow$ (2), (Q) $\rightarrow$ (1), (R) $\rightarrow$ (2), (S) $\rightarrow$ (1)
(b) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (1), (R) $\rightarrow$ (2), (S) $\rightarrow$ (3)
(c) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (3), (R) $\rightarrow$ (2), (S) $\rightarrow$ (5)
(d) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (1), (S) $\rightarrow$ (5)

16. Given system of equations $x + y + 2z = a, -2x - z = b, x + 3y + 5z = c$, (a, b, c) is an ordered triplet.
Match each entry in **List-I** to the correct entries in **List-II**.

List-I		List-II	
(P)	(0, 0, 0)	(1)	no solution
(Q)	(–1, 1, 1)	(2)	infinite solution
(R)	(1, 1, 4)	(3)	non-trivial solution
(S)	(2, 1, –1)	(4)	trivial solution
		(5)	exactly two solutions

The correct option is

(a) (P) $\rightarrow$ (2), (Q) $\rightarrow$ (1), (R) $\rightarrow$ (2), (S) $\rightarrow$ (1)
(b) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (1), (R) $\rightarrow$ (2), (S) $\rightarrow$ (3)
(c) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (4), (S) $\rightarrow$ (1)
(d) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (1), (S) $\rightarrow$ (5)

17. From a rectangular sheet of sides a and b, four equal squares of side x are cut off from the four corners. It is folded into an open rectangular box. **List-I** gives values of a and b and **List-II** gives values of x when the volume of the box is maximum.
Match each entry in **List-I** to the correct entries in **List-II**.

List-I		List-II	
(P)	$a = 7, b = 5$	(1)	$2 - \sqrt{\frac{7}{3}}$
(Q)	$a = 8, b = 4$	(2)	$2 - \frac{\sqrt{7}}{2}$
(R)	$a = 9, b = 3$	(3)	$2 - \frac{2}{\sqrt{3}}$
(S)	$a = 10, b = 2$	(4)	$2 - \frac{1}{2}\sqrt{\frac{13}{3}}$
		(5)	$2 - \sqrt{3}$

The correct option is

(a) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (3), (R) $\rightarrow$ (2), (S) $\rightarrow$ (1)
(b) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (3), (R) $\rightarrow$ (1), (S) $\rightarrow$ (1)
(c) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (1), (R) $\rightarrow$ (3), (S) $\rightarrow$ (5)
(d) (P) $\rightarrow$ (5), (Q) $\rightarrow$ (3), (R) $\rightarrow$ (2), (S) $\rightarrow$ (1)

PAPER-II

SECTION 1 (MAXIMUM MARKS: 12)

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +3 *If ONLY the correct option is chosen;*
Zero Marks : 0 *If none of the options is chosen (i.e. the question is unanswered);*
Negative Marks : –1 *In all other cases.*

1. There are four seats numbered 1, 2, 3, 4 in a room and four persons having tickets corresponding to these seats (one person having one ticket). Now the person having the ticket number 1, enters into the room and sits on any of the seat at random. Then the person having the ticket number 2, enters in room. If his seat is empty then he sits on his seat otherwise he sits on any of the empty seat at random. Similarly the other persons sit. Probability that the person having ticket numbered 4 gets the seat number 4 is

(a) 1/2 (b) 1/4 (c) 1/8 (d) 1/16

2. Consider $\int_0^x (t^2-8t+13)\,dt = x\sin\left(\frac{a}{x}\right)$ and $(a, x \in R - \{0\})$. x takes the values for which the equation has a solution, then the number of values of $a \in [0, 100]$ is

(a) 1 (b) 2 (c) 3 (d) 4

3. Sum of the series ${}^nC_1 \sin a + {}^nC_2 \sin 2a + ... + {}^nC_n \sin na$ is equal to

(a) $2^n \cos^n\left(\frac{a}{2}\right)\cdot\sin\left(\frac{na}{2}\right)$

(b) $2^n \sin^n\left(\frac{a}{2}\right)\cdot\cos\left(\frac{na}{2}\right)$

(c) $2^n \sin^n\left(\frac{a}{2}\right)\cdot\sin\left(\frac{na}{2}\right)$

(d) $2^n \cos^n\left(\frac{a}{2}\right)\cdot\cos\left(\frac{na}{2}\right)$

4. If $\frac{\pi a}{2} + 2(a+b) = L$, a constant, then the function $\frac{\pi a^2}{8} + 3ab$ is maximum when $\frac{b}{a} =$

(a) $1+\pi/3$ (b) $1-\pi/3$ (c) $1+\pi/6$ (d) $1-\pi/6$

SECTION 2 (MAXIMUM MARKS : 12)

- ***This section contains THREE (03) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).***
- ***For each question, choose the option(s) corresponding to (all) the correct answer(s).***
- ***Answer to each question will be evaluated according to the following marking scheme:***

Full Marks	:	***+4***	***ONLY if (all) the correct option(s) is(are) chosen;***
Partial Marks	:	***+3***	***If all the four options are correct but ONLY three options are chosen;***
Partial Marks	:	***+2***	***If three or more options are correct but ONLY two options are chosen, both of which are correct;***
Partial Marks	:	***+1***	***If two or more options are correct but ONLY one option is chosen and it is a correct option;***
Zero Marks	:	***0***	***If unanswered;***
Negative Marks	:	***–2***	***In all other cases.***

5. If $\int \frac{\log_x e \cdot \log_{ex} e \cdot \log_{e^2 x} e}{x}\,dx = A\log_e \log_e x + B\log_e(1+\log_e x) + C\log_e(2+\log_e x) + \lambda$, then

(a) $A + B = 0$ (b) $A + B = -1/2$

(c) $A + C = 0$ (d) $A - C = 0$

6. Let three terms of a non-constant positive G.P. be the sides of a triangle. If r is the common ratio of the G.P., then

(a) $1 < r < 2\cos 36°$ (b) $2\sin 18° < r < 1$

(c) $r^2 + r^{-2} = 4$ (d) none of these

7. The volume of the parallelepiped whose coterminous edges are represented by the vectors $2\vec{b}\times\vec{c}$, $3\vec{c}\times\vec{a}$ and $4\vec{a}\times\vec{b}$, where

$\vec{a} = (1+\sin\theta)\hat{i} + \cos\theta\,\hat{j} + \sin 2\theta\hat{k}$,

$\vec{b} = \sin\left(\theta+\frac{2\pi}{3}\right)\hat{i} + \cos\left(\theta+\frac{2\pi}{3}\right)\hat{j} + \sin\left(2\theta+\frac{4\pi}{3}\right)\hat{k}$,

$\vec{c} = \sin\left(\theta-\frac{2\pi}{3}\right)\hat{i} + \cos\left(\theta-\frac{2\pi}{3}\right)\hat{j} + \sin\left(2\theta-\frac{4\pi}{3}\right)\hat{k}$

is 18 cubic units, then the values of θ, in the interval $\left(0, \frac{\pi}{2}\right)$, is/are

(a) $\frac{\pi}{9}$ (b) $\frac{2\pi}{9}$ (c) $\frac{\pi}{3}$ (d) $\frac{4\pi}{9}$

SECTION 3 (MAXIMUM MARKS : 24)

- ***This section contains SIX (06) questions.***
- ***The answer to each question is a NON-NEGATIVE INTEGER.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

Full Marks	:	***+4***	***If ONLY the correct integer is entered;***
Zero Marks	:	***0***	***In all other cases.***

8. If $f(x+y+z) = f(x) + f(y) + f(z)$ with $f(1) = 1$ and $f(2) = 2$ and $x, y, z \in N$, then $\lim\limits_{n\to\infty} \frac{\sum\limits_{r=1}^{n}(4r)\,f(3r)}{n^3}$ is equal to _____.

9. If the number of ordered pairs of (x, y) satisfying the system of equations $5x\left(1+\frac{1}{x^2+y^2}\right)=12$ and $5y\left(1-\frac{1}{x^2+y^2}\right)=4$ is n, then n is _____.

10. Let $z = \frac{-1+\sqrt{3}i}{2}$, where $i=\sqrt{-1}$, and $r, s \in \{1, 2, 3\}$. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is _____.

11. If $y = y(x)$ satisfies the differential equation

$$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1}dx,\ x>0$$

and $y(0)=\sqrt{7}$, then $y(256)=$ _____ .

12. Let $f:(0,\infty)\to R$ and $F(x)=\int_0^x f(t)\,dt$.

If $F(x^2) = x^2\,(1+x)$, then $f(4)$ equals to_____ .

13. If $\vec{u},\vec{v},\vec{w}$, are three non-coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ and $\vec{v}$, $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$, respectively, and $\vec{x},\vec{y},\vec{z}$ are unit vectors along the bisectors of the angles α, β, γ, respectively.

If $[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}] = 1/\lambda\ \ [\vec{u}\ \vec{v}\ \vec{w}]^2 \sec^2\frac{\alpha}{2}$

$\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$, then the value of $\sqrt{\lambda}$ is _____ .

SECTION 4 (MAXIMUM MARKS : 12)

- ***This section contains TWO (02) paragraphs.***
- ***Based on each paragraph, there are TWO (02) questions.***
- ***The answer to each question is a NUMERICAL VALUE.***
- ***For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.***
- ***If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

Full Marks	:	***+3***	***If ONLY the correct numerical value is entered in the designated place;***
Zero Marks	:	***0***	***In all other cases.***

PARAGRAPH-I

Consider $f(x)=\begin{cases}[x], & -2\le x\le -\frac{1}{2}\\ 2x^2-1, & -\frac{1}{2}<x\le 2\end{cases}$

and $g(x) = f(|x|) + |f(x)|$, where $[\cdot]$ represents the greatest integer function.

Following are two questions based on PARAGRAPH-I.

14. The number of points where $|f(x)|$ is non-differentiable is _____ .

15. The number of points where $g(x)$ is non-differentiable is _____ .

PARAGRAPH-II

Let a, b and c be three real numbers satisfying

$$[a\ \ b\ \ c]\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix}=[0\ \ 0\ \ 0].$$

Following are two questions based on PARAGRAPH-II.

16. If the point $P(a, b, c)$, with reference to (i), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is _____ .

17. Let ω be a solution of $x^3 - 1 = 0$ with $lm(\omega) > 0$, if $a = 2$ with b and c satisfying (i), then the value of $\left|\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c}\right|$ is equal to _____ .

SOLUTIONS

PAPER-I

1. (c) : Let p be the probability of getting head when a coin is thrown once and q be the probability of not getting head

$$\therefore\ p=\frac{1}{2}\text{ and }q=1-p=1-\frac{1}{2}=\frac{1}{2}$$

Let X be the random variable denoting the number of heads when a coin is tossed $2n$ times.

Required probability = 1 – Probability of getting equal number of heads and tails = $1 - P(X = n)$

$$=1-{}^{2n}C_n\left(\frac{1}{2}\right)^{2n-n}\left(\frac{1}{2}\right)^n=1-{}^{2n}C_n\left(\frac{1}{2}\right)^n\cdot\left(\frac{1}{2}\right)^n$$

$$=1-{}^{2n}C_n\left(\frac{1}{2}\right)^{2n}=1-\frac{(2n)!}{n!n!}\cdot\left(\frac{1}{4}\right)^n=1-\frac{(2n)!}{(n!)^2}\cdot\frac{1}{4^n}$$

2. (a) : We have,

$$C=AB=\begin{bmatrix}x & 1 & 2\\ 3x & -1 & \frac{1}{4x^2+1}\end{bmatrix}\begin{bmatrix}\frac{1}{x^2} & \frac{1}{x}\\ 2x & 2\\ 3 & x\end{bmatrix}$$

$$\therefore\ \Delta(x)=\sum_{1\le i\le j\le 2} c_{ij}=c_{11}+c_{12}+c_{22}$$

$$=\left(\frac{1}{x}+2x+6\right)+(1+2+2x)+\left(3-2+\frac{x}{4x^2+1}\right)$$

$$=\frac{1}{x}+4x+10+\frac{1}{4x+\frac{1}{x}}$$

Let $\frac{1}{x}+4x=t\Rightarrow t\ge 4$

$$\therefore\ \Delta(x)|_{\min}=4+10+\frac{1}{4}=\frac{57}{4}$$

3. (c) : Given, 10 $\begin{cases} x \text{ black balls} \\ 10-x \text{ red balls} \end{cases}$

$$\therefore \quad \frac{{}^{x}C_1 \cdot {}^{10-x}C_1}{{}^{10}C_2} = \frac{8}{15} \Rightarrow \frac{x(10-x)}{45} = \frac{8}{15}$$

$\Rightarrow x^2 - 10x + 24 = 0$

$\Rightarrow x = 6$ or $x = 4$

Since, given that number of black balls is more than red balls

$\therefore$ Number of black balls $(BB) = 6$ and Number of red balls $(RB) = 4$

Now, $P(E) = 1 - P(RR) = 1 - \frac{{}^{4}C_2}{{}^{10}C_2} = \frac{39}{45}$

4. (c) : Let $L = \lim\limits_{n\to\infty} \frac{1}{m}\left(\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdots\frac{n}{n}\right)^{1/n}$

Taking logarithms on bath sides, we get

$$\Rightarrow \quad \ln L = \lim_{n\to\infty}\left[\ln\left(\frac{1}{m}\right) + \frac{1}{n}\left(\ln\frac{1}{n} + \ln\frac{2}{n} + \ldots\ldots + \ln\frac{n}{n}\right)\right]$$

$$= -\ln m + \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(\frac{r}{n}\right)$$

$$= -\ln m + \int_0^1 \ln x\,dx = -\ln m - 1 = \ln\left(\frac{1}{em}\right) \therefore L = \frac{1}{em}$$

5. (a, b, c, d)

6. (c, d) : $\tan^{-1}\frac{a}{x} + \tan^{-1}\frac{b}{x} = \frac{\pi}{2} - \left(\tan^{-1}\frac{c}{x} + \tan^{-1}\frac{d}{x}\right)$

$$\Rightarrow \quad \tan^{-1}\left(\frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{\frac{c}{x}+\frac{d}{x}}{1-\frac{cd}{x^2}}\right)$$

$$\Rightarrow \quad \tan^{-1}\frac{(a+b)x}{x^2-ab} = \cot^{-1}\frac{(c+d)x}{x^2-cd}$$

$$\Rightarrow \quad \frac{(a+b)x}{x^2-ab} = \frac{x^2-cd}{(c+d)x}$$

$$\Rightarrow \quad x^4 - (ab+ac+ad+bc+bd+cd)x^2 + abcd = 0$$

Its roots are x_1, x_2, x_3, x_4

$$\therefore \quad \sum x_1 = 0, \ \sum x_1x_2 = -\sum ab, \ \sum x_1x_2x_3 = 0$$

and $x_1x_2x_3x_4 = abcd$

$$\therefore \quad \sum\frac{1}{x_1} = 0 \text{ and}$$

$(x_2 + x_3 + x_4)(x_3 + x_4 + x_1)\ (x_4 + x_1 + x_2)\ (x_1 + x_2 + x_3)$

$= x_1x_2x_3x_4 = abcd$

7. (a, b, d) : Any point on the line through $P(\lambda, 2)$ is $(\lambda + r\cos\theta, 2 + r\sin\theta)$ and meets the ellipse at A and D

So, $\frac{(\lambda + r\cos\theta)^2}{9} + \frac{(2 + r\sin\theta)^2}{4} = 1$

So, $r_1r_2 = PA \cdot PD = \frac{4\lambda^2}{4\cos^2\theta + 9\sin^2\theta}$

Again solving with $x = 0$ and $y = 0$, we have

$$PB \cdot PC = \frac{2\lambda}{\sin\theta\cos\theta}$$

So, $\frac{4\lambda^2}{4\cos^2\theta + 9\sin^2\theta} = \frac{2\lambda}{\sin\theta\cos\theta}$

i.e., $2\lambda\sin2\theta + 5\cos2\theta = 13$

So, $-1 \le \frac{13}{\sqrt{4\lambda^2+25}} \le 1$ *i.e.*, $|\lambda| \ge 6$

8. (3) : We have, $z = re^{i\theta} \Rightarrow e^{2i\theta} + e^{i\theta} + 1 = 0$

$\Rightarrow \cos 2\theta + \cos\theta + 1 = 0, \sin 2\theta + \sin\theta = 0$

$$\Rightarrow \quad \theta = \pm\frac{2\pi}{3}, \ z_1 = re^{\frac{i2\pi}{3}} = r\omega$$

$z_2 = re^{\frac{-2\pi i}{3}} = r\omega^2$, where ω is a complex cube root of unity Now, $|z_1 - z_2|^2 = r^2\,|\omega - \omega^2|^2$

$$= r^2\left|-\frac{1}{2} + \frac{i\sqrt{3}}{2} + \frac{1}{2} + \frac{i\sqrt{3}}{2}\right|^2 = r^2\,|i\sqrt{3}\,|^2 = 3r^2$$

9. (2) : We have,

$\alpha^7 = 1 \Rightarrow 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6 = 0$

Sum of the roots, $\alpha + \alpha^2 + \alpha^4 + \alpha^3 + \alpha^5 + \alpha^6 = -1$

and c = product of the roots $= (\alpha + \alpha^2 + \alpha^4)(\alpha^3 + \alpha^5 + \alpha^6)$

$= 3 + \alpha^4 + \alpha^5 + \alpha^6 + \alpha^2 + \alpha + \alpha^3 = 3 - 1 = 2$

10. (4) : We have, $N = (1-2)^2 + (1-3)^2 + \ldots\ldots + (1-20)^2$

$+ (2-3)^2 + \ldots\ldots\ldots + (2-20)^2 \ldots\ldots + (19-20)^2$

$$= 19\,(1^2 + 2^2 + \ldots + 20^2) - 2\sum_{1\le r<s\le 20} rs$$

$= 19\,(1^2 + 2^2 + \ldots + 20^2) - (1 + 2 + \ldots\ldots + 20)^2 +$

$(1^2 + 2^2 + \ldots\ldots + 20^2)$

$= 20(1^2 + 2^2 + \ldots + 20^2) - (1 + 2 + \ldots\ldots\ldots + 20)^2$

$$= \frac{20\cdot 20\cdot 21\cdot 41}{6} - \frac{20^2\cdot 21^2}{4}$$

$= (574 - 441) \times 100 = 13300 = 2^2\cdot 5^2\cdot 7\cdot 19.$

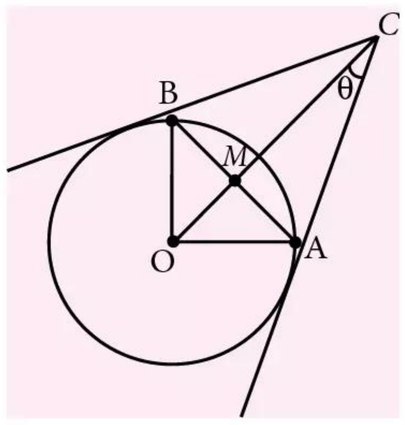
11. (1) : Since, $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n} = 2^n$

$$\therefore \quad 19\ N = \frac{2^{19} - 2 - 38}{2} = 2^{18} - 20$$

Now, $\frac{N}{10^4} = \frac{2^{16} - 5}{19 \times 2500} = \frac{65531}{47500} = 1.379$

12. (3) : We have, $M \equiv (2, 3) \Rightarrow OM = \sqrt{13}$

$OA = 5 \Rightarrow MA = \sqrt{25 - 13} = 2\sqrt{3}$



Area of kite $OACB$
$= 2$ area of ΔOAC
$= OA \cdot AC = 5 \times 5 \cot\theta$

$= 25 \cdot \dfrac{2\sqrt{3}}{\sqrt{13}} = 50\sqrt{\dfrac{3}{13}}$

13. (9) : Required number of ways
$= P(4, 3) + P(3, 2) + P(2, 1)$

$= \dfrac{4!}{3!1!} + \dfrac{3!}{2!1!} + \dfrac{2!}{1!1!} = 4 + 3 + 2 = 9$

14. (a) : (P) We have, $r_3 = s\tan\dfrac{C}{2} = s\tan\dfrac{\pi}{4} = s$

$\Delta^2 = r_1 r_2 r_3 r = r_1 r_2 sr = r_1 r_2 \Delta$

$\therefore \quad \Delta = r_1 r_2 = 11 \times 90$

$= 2^1 \cdot 3^2 \cdot 5^1 \cdot 11^1$, which has 24 divisors.

(Q) $\dfrac{\Delta}{r} = \dfrac{\Delta}{r_1} + \dfrac{\Delta}{r_2} + \dfrac{\Delta}{r_3} \Rightarrow s = 90 + 11 + \dfrac{11 \times 90}{s}$

$\Rightarrow \quad s^2 - 101s - 990 = 0 \Rightarrow \; s = 110$

$\therefore \quad r_3 = s = 2^1 \cdot 5^1 \cdot 11^1$, which has 8 divisors

(R) $s - a = \dfrac{\Delta}{r_1} = r_2 \Rightarrow a = s - r_2 = 110 - 90 = 20 = 2^2 \cdot 5^1$,
which has 6 divisors

(S) $s - b = \dfrac{\Delta}{r_2} = r_1 \Rightarrow b = s - r_1 = 110 - 11 = 99 = 3^2 \cdot 11^1$,
which has 6 divisors.

15. (c) : Here $L \equiv \left(1, \dfrac{1}{\sqrt{2}}\right)$, The tangent at L is
$x + \sqrt{2}y = 2$

The normal at L is $\sqrt{2}x - y = \dfrac{1}{\sqrt{2}}$

$\therefore \quad T \equiv (2, 0), N \equiv (1/2, 0), \; M \equiv \left(-2, \dfrac{-5}{\sqrt{2}}\right)$

(P) Area of $\Delta LNT = \dfrac{3}{4\sqrt{2}}$

(Q) Circumradius $= \dfrac{NT}{2} = \dfrac{3}{4}$

(R) $LN : LT = \dfrac{1}{\sqrt{2}}$

(S) $\dfrac{LN}{NM} = \dfrac{1}{5}$

16. (a) : Here, $\Delta = 0$ and $\Delta_x = \Delta_y = \Delta_z = 0$
$\Rightarrow \quad 3a + b - c = 0$

17. (a) : $V = x(a - 2x)(b - 2x) = 4x^3 - 2(a + b)x^2 + abx$,

$\dfrac{dV}{dx} = 0 \Rightarrow 4x^2 - \dfrac{4}{3}(a+b)x + \dfrac{ab}{3} = 0$

$\therefore \quad x = \dfrac{a+b \pm \sqrt{a^2 - ab + b^2}}{6}$

(P) $a = 7, b = 5 \Rightarrow x = 2 \pm \dfrac{\sqrt{39}}{6} = 2 - \dfrac{1}{2}\sqrt{\dfrac{13}{3}}$

or $2 + \dfrac{1}{2}\sqrt{\dfrac{13}{3}}$ (Rejected)

(Q) $a = 8, b = 4 \Rightarrow x = 2 \pm \dfrac{2}{\sqrt{3}} = 2 - \dfrac{2}{\sqrt{3}}$ or $2 + \dfrac{2}{\sqrt{3}}$ (Rejected)

(R) $a = 9, b = 3 \Rightarrow x = 2 \pm \dfrac{\sqrt{7}}{2} = 2 - \dfrac{\sqrt{7}}{2}$ or $2 + \dfrac{\sqrt{7}}{2}$ (Rejected)

(S) $a = 10, b = 2 \Rightarrow x = 2 \pm \sqrt{\dfrac{7}{3}} = 2 - \sqrt{\dfrac{7}{3}}$ or $2 + \sqrt{\dfrac{7}{3}}$ (Rejected)

PAPER-II

1. (a) : Way 1 : 1 take 1, 2 take 2, 3 take 3, and 4 take 4

$\therefore \quad P(\text{way 1}) = \dfrac{1}{4} \times 1 \times 1 \times 1 = \dfrac{1}{4}$

Way 2 : 1 take 2, 2 take 1, 3 take 3, 4 take 4

$\therefore \quad P(\text{way 2}) = \dfrac{1}{4} \times \dfrac{1}{3} \times 1 \times 1 = \dfrac{1}{12}$

Way 3 : 1 take 2, 2 take 3, 3 take 1, 4 take 4

$\therefore \quad P(\text{way 3}) = \dfrac{1}{4} \times \dfrac{1}{3} \times \dfrac{1}{2} \times 1 = \dfrac{1}{24}$

Way 4 : 1 take 3, 2 take 2, 3 take 1, 4 take 4

$\therefore \quad P(\text{way 4}) = \dfrac{1}{4} \times 1 \times \dfrac{1}{2} \times 1 = \dfrac{1}{8}$

Required probability $= \dfrac{1}{4} + \dfrac{1}{12} + \dfrac{1}{24} + \dfrac{1}{8} = \dfrac{1}{2}$

2. (c) : $\left(\dfrac{t^3}{3} - \dfrac{8t^2}{2} + 13t\right)_0^x = x\sin\left(\dfrac{a}{x}\right)$

$\Rightarrow \quad \dfrac{1}{3}(x^2 - 12x + 39) = \sin\left(\dfrac{a}{x}\right)$

Here $x \neq 0 \;\Rightarrow \; \dfrac{1}{3}(x^2 - 12x + 39) = \sin\left(\dfrac{a}{x}\right)$

$\Rightarrow \quad \dfrac{1}{3}(x-6)^2 + 1 = \sin\left(\dfrac{a}{x}\right) \Rightarrow \dfrac{a}{6} = \dfrac{\pi}{2}$ or $\dfrac{5\pi}{2}$ or $\dfrac{9\pi}{2}$

$\therefore \quad a = 3\pi, 15\pi, 27\pi$ (3 values)

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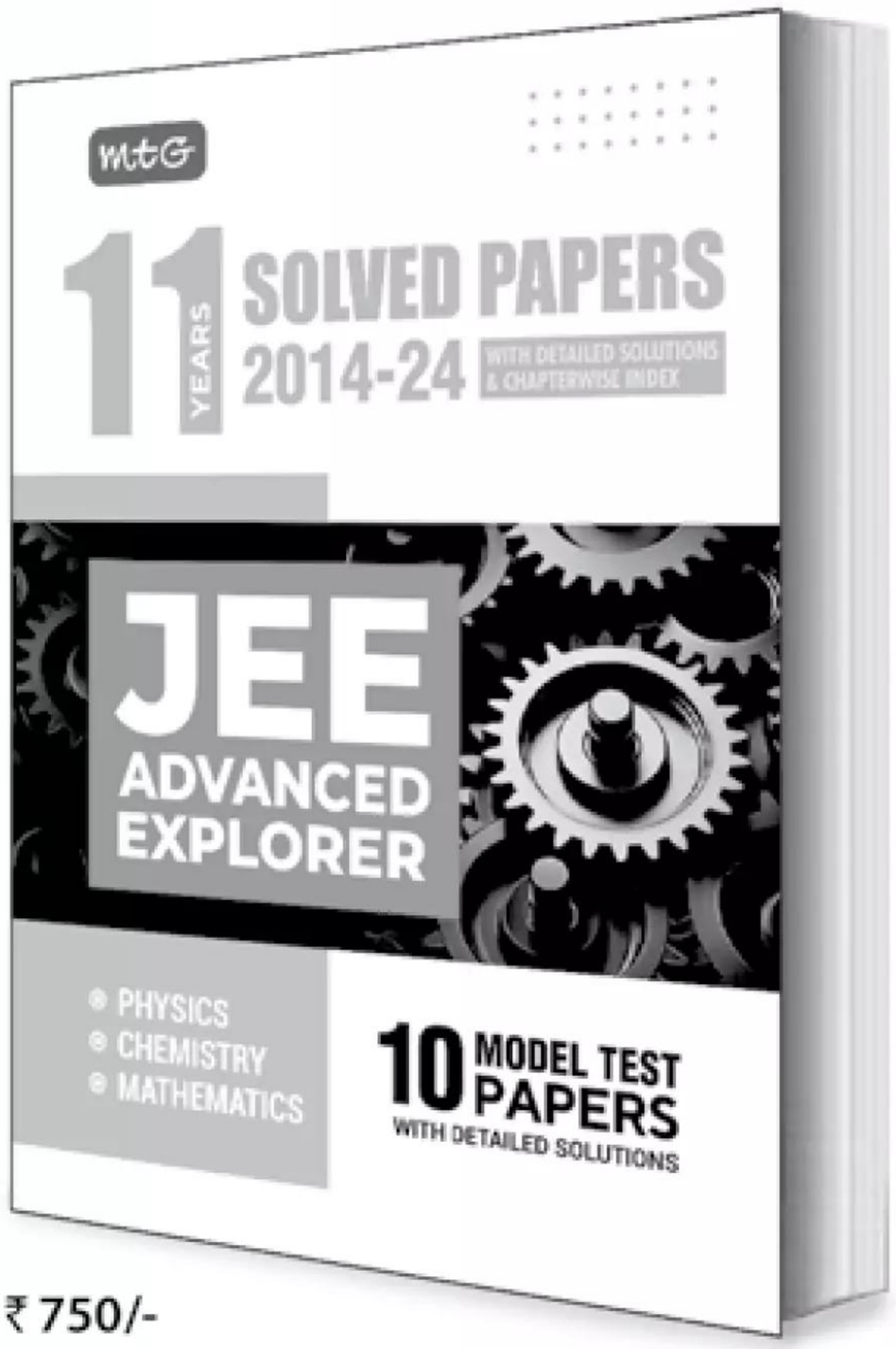

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3. (a) : $S = {}^nC_1 \sin a + {}^nC_2 \sin 2a + ... + {}^nC_n \sin na$

and $C = {}^nC_0 + {}^nC_1 \cos a + {}^nC_2 \cos 2a + ... + {}^nC_n \cos na$

$\Rightarrow \quad C + iS = 1 + {}^nC_1(\cos a + i\sin a) + {}^nC_2(\cos 2a + i\sin 2a)$
$+ ... + {}^nC_n(\cos na + i\sin na)$

$\Rightarrow \quad C + iS = (1 + z)^n$, where $z = \cos a + i \sin a$

$$= \left\{2\cos\frac{a}{2}\left(\cos\frac{a}{2} + i\sin\frac{a}{2}\right)\right\}^n$$

$$\Rightarrow \quad S = 2^n \cos^n\left(\frac{a}{2}\right)\cdot\sin\left(\frac{na}{2}\right)$$

4. (c)

5. (b, d) : Let $I = \int \frac{\log_x e \cdot \log_{ex} e \cdot \log_{e^2 x} e}{x}\, dx$

$$= \int \frac{dx}{x \log_e x\,(1 + \log_e x)\,(2 + \log_e x)}$$

Put $\log_e x = t \quad \frac{dx}{x} = dt$

$$\Rightarrow \quad I = \int \frac{dt}{t(1 + t)\,(2 + t)}$$

Now, $\frac{1}{t(1+t)(2+t)} = \frac{A}{t} + \frac{B}{1+t} + \frac{C}{2+t}$

$$\Rightarrow \quad A = \frac{1}{2}, B = -1, C = \frac{1}{2}$$

$$\therefore \quad I = \frac{1}{2}\int \frac{1}{t}dt - 1\int \frac{1}{1+t}dt + \frac{1}{2}\int \frac{dt}{2+t}$$

$$= \frac{1}{2}\log_e t - \log_e(1 + t) + \frac{1}{2}\log_e(2 + t)$$

$$= \frac{1}{2}\log_e \log_e x - \log_e(1 + \log_e x) + \frac{1}{2}\log_e(2 + \log_e x)$$

6. (a, b) : If $r > 1$, then ar^2 is the greatest term.

$\therefore \quad ar^2 < a + ar \Rightarrow r^2 - r - 1 < 0$

$$\Rightarrow \quad \frac{1}{2}(1 - \sqrt{5}) < r < \frac{1}{2}(1 + \sqrt{5})$$

As $r > 1$, $1 < r < \frac{1}{2}(\sqrt{5} + 1) = 2\cos 36°$

In case $0 < r < 1$, $a < ar + ar^2 \Rightarrow r^2 + r - 1 > 0$

$$\Rightarrow \quad r < -\frac{1}{2}(\sqrt{5} + 1) \text{ or } r > \frac{1}{2}(\sqrt{5} - 1)$$

Thus, $2 \sin 18° < r < 1$

7. (a, b, d) : Given,

Volume $= \left|[2\vec{b} \times \vec{c} \quad 3\vec{c} \times \vec{a} \quad 4\vec{a} \times \vec{b}]\right| = 18$

$$\Rightarrow \quad 24[\vec{a}\ \vec{b}\ \vec{c}]^2 = 18 \Rightarrow \left|[\vec{a}\ \vec{b}\ \vec{c}]\right| = \frac{\sqrt{3}}{2}$$

Now, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} (1+\sin\theta) & \cos\theta & \sin 2\theta \\ \sin\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(2\theta + \frac{4\pi}{3}\right) \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$ and expanding

$$\left|[\vec{a}\ \vec{b}\ \vec{c}]\right| = \sqrt{3}\left|\cos 3\theta\right| \Rightarrow \frac{\sqrt{3}}{2} = \sqrt{3}\,|\cos 3\theta|$$

$$\Rightarrow \quad \cos 3\theta = \pm\frac{1}{2} \Rightarrow 3\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3} \Rightarrow \theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}$$

8. (4) : $f(3) = 3f(1) = 3, f(4) = f(2+1+1)$
$= 2+1+1 = 4$ and so on.

In general, we get $f(r) = r$ for $r \in N$.

$$\Rightarrow \quad \lim_{n\to\infty} \frac{\sum_{r=1}^{n}(4r)\,f(3r)}{n^3} = \lim_{n\to\infty} \frac{12n(n+1)(2n+1)}{6n^3} = 4$$

9. (2) : $5x\left(1 + \frac{1}{x^2 + y^2}\right) = 12 \quad \Rightarrow \quad x = \frac{12}{5}\frac{1}{1 + \frac{1}{x^2 + y^2}}$ ...(i)

Also, $5y\left(1 - \frac{1}{x^2 + y^2}\right) = 4 \Rightarrow y = \frac{4}{5}\cdot\frac{1}{1 - \frac{1}{x^2 + y^2}}$ ...(ii)

Squaring (i) and (ii) and then adding, we have

$$x^2 + y^2 = \left(\frac{4}{5}\right)^2\left\{\frac{9}{\left(1 + \frac{1}{x^2 + y^2}\right)^2} + \frac{1}{\left(1 - \frac{1}{x^2 + y^2}\right)^2}\right\}$$

$$\Rightarrow \quad \frac{t}{t^2} = \left(\frac{4}{5}\right)^2\left\{\frac{9}{(t+1)^2} + \frac{1}{(t-1)^2}\right\}, \text{ where } x^2 + y^2 = t$$

$$\Rightarrow \quad \frac{1}{t} = \left(\frac{4}{5}\right)^2\left\{\frac{10t^2 - 16t + 10}{(t^2 - 1)^2}\right\}$$

$$\Rightarrow \quad 25(t^4 - 2t^2 + 1) = 16t(10t^2 - 16t + 10)$$

$$\Rightarrow \quad 25t^2\left(t^2 + \frac{1}{t^2} - 2\right) = 16(2t^2)\left\{5t + \frac{5}{t} - 8\right\}$$

$$\Rightarrow \quad 25t^2\left(\left(t + \frac{1}{t}\right)^2 - 4\right) = 32t^2\left\{5\left(t + \frac{1}{t}\right) - 8\right\}$$

$$\Rightarrow \quad 25\{z^2 - 4\} = 32\{5z - 8\}, \text{ where } t + \frac{1}{t} = z$$

$$\Rightarrow \quad (5z - 6)(5z - 26) = 0 \quad \therefore \quad z = \frac{6}{5} \text{ or } z = \frac{26}{5}$$

$$\Rightarrow \quad t+\frac{1}{t}=\frac{6}{5} \quad \text{or} \quad t+\frac{1}{t}=5+\frac{1}{5}$$

$$\because \quad t+\frac{1}{t}\geq 2 \text{ so, either } t=5 \text{ or } \frac{1}{5}$$

$$\therefore \quad x^2+y^2=5 \quad \text{or} \quad x^2+y^2=\frac{1}{5}$$

$$\therefore \quad x=\frac{12}{5}\frac{1}{\left(1+\frac{1}{5}\right)} \quad \text{or} \quad x=\frac{12}{5}\left(\frac{1}{1+5}\right)$$

$$\Rightarrow \quad x=\frac{12}{5}\cdot\frac{5}{6}=2 \quad \text{or} \quad x=\frac{12}{5}\cdot\frac{1}{6}=\frac{2}{5}$$

$$\Rightarrow \quad y=1 \quad \text{or} \quad y=-\frac{1}{5}$$

Hence, $(x, y) = (2, 1)$ or $\left(\frac{2}{5}, -\frac{1}{5}\right)$

Hence, the given system possesses two solutions.

10. (1) : Note that $z = \omega$

$$P\equiv\begin{bmatrix}(-\omega)^r & \omega^{2s}\\ \omega^{2s} & \omega^r\end{bmatrix}$$

$$P^2=\begin{bmatrix}\omega^{2r}+\omega^{4s} & \omega^{r+2s}\{(-1)^r+1\}\\ \omega^{r+2s}\{(-1)^r+1\} & \omega^{4s}+\omega^{2r}\end{bmatrix}=-I$$

$\Rightarrow (-1)^r + 1 = 0 \quad \therefore \quad r = 1$ or 3

Again, $\omega^{2r} + \omega^{4s} = -1$. r and s both can't be 3.

$\therefore \quad r = 1$, then $s = 1 \ \therefore (r, s) = (1, 1)$ only.

11. (3) : Rewrite the differential equation as

$$\frac{dy}{dx}=\frac{1}{8\sqrt{x}\sqrt{9+\sqrt{x}}}\cdot\frac{1}{\sqrt{4+\sqrt{9+\sqrt{x}}}}$$

Let $P=\sqrt{4+\sqrt{9+\sqrt{x}}}$

Then, $\frac{dP}{dx}=\frac{1}{2\sqrt{4+\sqrt{9+\sqrt{x}}}}\cdot\frac{1}{2\sqrt{9+\sqrt{x}}}\cdot\frac{1}{2\sqrt{x}}$

Now, we have $\frac{dy}{dx}=\frac{dP}{dx}$. Then $y = P(x) + \lambda$

$\Rightarrow \quad y=\sqrt{4+\sqrt{9+\sqrt{x}}}+\lambda$. Now, $y(0)=\sqrt{7} \Rightarrow \lambda = 0$

Then, $y=\sqrt{4+\sqrt{9+\sqrt{x}}}$

$\therefore \quad$ At $x=256$, $y=\sqrt{4+\sqrt{9+\sqrt{256}}}=\sqrt{4+\sqrt{9+16}}=3$

12. (4) : $F'(x) = f(x)$

Now, $F(x^2) = x^2(1 + x) \Rightarrow 2xF'(x^2) = 2x + 3x^2$

Putting $x = 2$, we get $2 \cdot 2F'(4) = 2 \times 2 + 3 \cdot 2^2$

$\Rightarrow \quad F'(4) = 4 \quad \therefore \quad f(4) = 4$

13. (4) : We know that $[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}]=[\vec{x}\,\vec{y}\,\vec{z}]^2$

Also a vector along the bisector of given two unit vectors $\vec{u}, \vec{v}$ is $\vec{u}+\vec{v}$.

A unit vector along the bisector is $\frac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|}$.

$$|\vec{u}+\vec{v}|^2=1+1+2\vec{u}\cdot\vec{v}=2+2\cos\alpha=4\cos^2\frac{\alpha}{2}$$

$$\Rightarrow \quad \vec{x}=\frac{\vec{u}+\vec{v}}{2\cos\frac{\alpha}{2}}$$

Similarly, $\vec{y}=\frac{\vec{v}+\vec{w}}{2\cos(\beta/2)}$ and $\vec{z}=\frac{\vec{u}+\vec{w}}{2\cos(\gamma/2)}$

$$\Rightarrow [\vec{x}\,\vec{y}\,\vec{z}]=\frac{1}{8}[\vec{u}+\vec{v}\ \ \vec{v}+\vec{w}\ \ \vec{u}+\vec{w}]\times\sec\frac{\alpha}{2}\sec\frac{\beta}{2}\sec\frac{\gamma}{2}$$

$$=\frac{1}{8}2[\vec{u}\,\vec{v}\,\vec{w}]\sec\frac{\alpha}{2}\sec\frac{\beta}{2}\sec\frac{\gamma}{2}=\frac{1}{4}[\vec{u}\,\vec{v}\,\vec{w}]\sec\frac{\alpha}{2}\sec\frac{\beta}{2}\sec\frac{\gamma}{2}$$

Now, $[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}]=[\vec{x}\,\vec{y}\,\vec{z}]^2$

$$=\frac{1}{16}[\vec{u}\,\vec{v}\,\vec{w}]^2\sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$$

(14 and 15) :

$$f(x)=\begin{cases}[x], & -2\leq x\leq -\frac{1}{2}\\ 2x^2-1, & -\frac{1}{2}<x\leq 2\end{cases}=\begin{cases}-2, & -2\leq x<-1\\ -1, & -1\leq x\leq -\frac{1}{2}\\ 2x^2-1, & -\frac{1}{2}<x\leq 2\end{cases}$$

$$|f(x)|=\begin{cases}2, & -2\leq x<-1\\ 1, & -1\leq x\leq -\frac{1}{2}\\ |2x^2-1|, & -\frac{1}{2}<x\leq 2\end{cases}=\begin{cases}2, & -2\leq x<-1\\ 1, & -1\leq x\leq -\frac{1}{2}\\ 1-2x^2, & -\frac{1}{2}<x\leq \frac{1}{\sqrt{2}}\\ 2x^2-1, & \frac{1}{\sqrt{2}}<x\leq 2\end{cases}$$

$$f(|x|)=\begin{cases}2x^2-1, & -2\leq x<-1\\ 2x^2-1, & -1\leq x\leq -\frac{1}{2}\\ 2x^2-1, & -\frac{1}{2}<x\leq 2\end{cases}$$

14. (3) : Let $h(x) = |f(x)|$

$$h(-1^-)=\lim_{x\to -1}(2)=2,\ h(-1^+)=\lim_{x\to -1}(1)=1$$

$$h\left(-\frac{1^-}{2}\right)=\lim_{x\to -\frac{1}{2}}(1)=1,\ h\left(-\frac{1^+}{2}\right)=\lim_{x\to \frac{-1}{2}}(1-2x^2)=\frac{1}{2}$$

$$h\left(\frac{1^-}{\sqrt{2}}\right)=\lim_{x\to \frac{1}{\sqrt{2}}}(1-2x^2)=0$$

$$h\left(\frac{1^+}{\sqrt{2}}\right)=\lim_{x\to\frac{1}{\sqrt{2}}}(2x^2-1)=0$$

$\therefore$ $h(x)$ is not continuous at $x=-1$ and $-1/2$

Hence, $h'(x)$ is not differentiable at $x=-1$ and $-1/2$

$$h'(x)=\begin{cases}0, & -2\le x<-1\\ 0, & -1\le x\le\dfrac{-1}{2}\\ -4x, & \dfrac{-1}{2}<x\le\dfrac{1}{\sqrt{2}}\\ 4x, & \dfrac{1}{\sqrt{2}}<x\le 2\end{cases}$$

$$h'\left(\frac{1^-}{\sqrt{2}}\right)=\frac{-4}{\sqrt{2}}\text{ and }h'\left(\frac{1^+}{\sqrt{2}}\right)=\frac{-4}{\sqrt{2}}$$

$\therefore$ $h(x)$ is not differentiable at $x=\dfrac{1}{\sqrt{2}}$

15. (3) : $\therefore$ $g(x)=f(|x|)+|f(x)|$

$$=\begin{cases}2x^2+1, & -2\le x<-1\\ 2x^2, & -1\le x\le-\dfrac{1}{2}\\ 0, & -\dfrac{1}{2}<x<\dfrac{1}{\sqrt{2}}\\ 4x^2-2, & \dfrac{1}{\sqrt{2}}\le x\le 2\end{cases}$$

$$g(-1^-)=\lim_{x\to-1}(2x^2+1)=3,\ g(-1^+)=\lim_{x\to-1}2x^2=2$$

$$g\left(-\frac{1^-}{2}\right)=\lim_{x\to-\frac{1}{2}}2x^2=\frac{1}{2},\ g\left(-\frac{1^+}{2}\right)=\lim_{x\to-\frac{1}{2}}0=0$$

$$g\left(\frac{1^-}{\sqrt{2}}\right)=\lim_{x\to\frac{1}{\sqrt{2}}}0=0,\ g\left(\frac{1^+}{\sqrt{2}}\right)=\lim_{x\to\frac{1}{\sqrt{2}}}(4x^2-2)=0$$

Hence, $g(x)$ is discontinuous at $x=-1,\ \dfrac{1}{2}$.

$g(x)$ is continuous at $x=\dfrac{1}{\sqrt{2}}$.

Now, $g'\left(\dfrac{1^-}{\sqrt{2}}\right)=0,\ g'\left(\dfrac{1^+}{\sqrt{2}}\right)=8\left(\dfrac{1}{\sqrt{2}}\right)=\dfrac{8}{\sqrt{2}}$

Hence, $g(x)$ is non-differentiable at $x=\dfrac{1}{\sqrt{2}}$.

(16 and 17) :

$a+8b+7c=0,\ 9a+2b+3c=0,\ a+b+c=0$

$$\Delta=\begin{vmatrix}1 & 8 & 7\\ 9 & 2 & 3\\ 1 & 1 & 1\end{vmatrix}=1(-1)-8(6)+7(7)=0$$

Let $c=\lambda$ $\therefore$ $a+8b=-7\lambda$ and $a+b=-\lambda$

$\Rightarrow$ $b=\dfrac{-6}{7}\lambda$ and $a=\dfrac{-\lambda}{7}$

$\therefore$ $(a,b,c)=\left(\dfrac{-\lambda}{7},\dfrac{-6\lambda}{7},\lambda\right)$ where $\lambda\in R$

16. (6) : $P(a, b, c)$ lies on the plane $2x+y+z=1$

$\dfrac{-2\lambda}{7}-\dfrac{6\lambda}{7}+\lambda=1\Rightarrow\dfrac{-\lambda}{7}=1\Rightarrow\lambda=-7$

$\therefore$ $7a+b+c=7+6-7=6.$

17. (2) : $a=2\Rightarrow\lambda=-14$

$\therefore$ $b=12$ and $c=-14$

Now, $\dfrac{3}{\omega^a}+\dfrac{1}{\omega^b}+\dfrac{3}{\omega^c}=\dfrac{3}{\omega^2}+\dfrac{1}{\omega^{12}}+3\omega^{14}$

$=3\omega+1+3\omega^2=3(\omega+\omega^2)+1=-2$

$\therefore$ $\left|\dfrac{3}{\omega^a}+\dfrac{1}{\omega^b}+\dfrac{3}{\omega^c}\right|=2$

EXAM ALERT 2025

CBSE Exam	Date
Physics Class 12	17th February
Chemistry Class 12	27th February
Physical Education Class 12	21st February
Mathematics Class 12	8th March
English Elective/Core Class 12	11th March
Hindi Elective/Core Class 12	15th March
Biology Class 12	25th March
Computer Science Class 12	29th March
Entrance Exam	**Date**
JEE Main Session 2	Between 1st April to 8th April
NDA I	13th April
K CET (PC)	16th April
K CET (MB)	17th April
VITEEE	21st- 27th April (Tentatively)
KEAM	Between 22nd April and 30th April
COMED K	10th May
JEE Advanced	18th May
NDA II	14th September
SRM JEEE	Phase 1 : 22nd April to 27th April Phase 2 : 12th June to 17th June Phase 3 : 4th July to 5th July
BITSAT Session-1	26th to 30th May, 2025
BITSAT Session-2	22nd to 26th June, 2025

Quiz CLUB

Unlock Your Knowledge!

1. Write the coefficient of y in the expansion of $\left(y^2+\frac{c}{y}\right)^5$.
2. Find the value of $\sqrt{-25}+3\sqrt{-4}+2\sqrt{9}$.
3. Find the perpendicular distance from the point $(3, -4)$ to the line $3x - 4y + 10 = 0$.
4. Find the value of $\lim\limits_{x\to 0}\frac{(\sin(a+x)-\sin(a-x))}{x}$.
5. Four dice are rolled. Find the number of possible outcomes in which at least one dice shows 2.
6. Find the projection of $\vec{a}$ on $\vec{b}$, if $\vec{a}\cdot\vec{b}=8$ and $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$.
7. Write the number of all the one-one functions from $A = \{1, 2\ 3\}$ to itself.
8. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix}=\begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$, then find the value of $a + b$.
9. If A and B are disjoint sets and A' is the complement of A, then find is the value of $B \cap A'$.
10. Find the value of $7! - 5!$.
11. If a complex number Z lies in the interior or on the boundary of a circle of radius 3 units and centre $(-4, 0)$, then find the greatest value of $|z + 1|$.
12. The side of an equilateral triangle is increasing at the rate of 2 cm/s. What is the rate at which area increases, when the side is 10 cm?
13. If $f(x) = x \sin x$, then find $f'\left(\frac{\pi}{2}\right)$.
14. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then find the value of $\cos^{-1}x + \cos^{-1}y$.
15. Find the area (in sq. units) bounded by the curves $y^2 = 4x$ and $y = x$.
16. If $\frac{d}{dx}f(x)=g(x)$, then what is the anti-derivative of $g(x)$?
17. Find the angle between the pair of lines $x^2 + 4xy + y^2 = 0$.
18. Find the eccentricity of the conic $x^2 - y^2 = (\sqrt{5})^{\frac{1}{10}}$.
19. If the value of a third order determinant is 7, then what will be the value of square of determinant formed by its cofactors?
20. Find the value of 'a' if the plane $2x - 3y + 6z - 11 = 0$, makes an angle $\sin^{-1}(a)$ with x–axis.

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.

SOLUTIONS TO FEBRUARY 2025 QUIZ CLUB

1. 4 **2.** $I - A$ **3.** $\frac{6}{11}$ **4.** 0 **5.** 1

6. (15, 5) **7.** 2 **8.** $\frac{-3}{\sqrt[3]{\sin x}}+C$

9. 0 **10.** $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$

11. $(-\infty, -2) \cup [6, \infty)$ **12.** $-\frac{1}{2}$ **13.** 0

14. $a < 9$ **15.** $b \in \left(-\frac{3}{4}, 3\right)$ **16.** $\sqrt{2}$

17. $I = \pi$ **18.** 8 **19.** ± 60 **20.** 4, 30°

PRACTICE PAPER 2025

Karnataka CET

1. If $A = \{x : x = 4n, n \in Z\}$ and $B = \{x : x = 6n, n \in Z\}$, then $A \cap B =$

(a) $\{x : x = 2n, n \in Z\}$ (b) $\{x : x = 10n, n \in Z\}$

(c) $\{x : x = 12n, n \in Z\}$ (d) $\{x : x = 24n, n \in Z\}$

2. Which one of the following is equal to $A \cup (A \cap B) \cup (B - A)$?

(a) B (b) $A \cup B$ (c) A (d) $A \cap B$

3. Let R be the relation on N defined as xRy iff $x + 4y = 16$. The domain of R is

(a) $\{4, 8\}$ (b) $\{4, 6, 8\}$

(c) $\{4, 8, 12\}$ (d) $\{1, 2, 3, 4\}$

4. If the domain of the function $f(x) = x^2 - 6x + 7$ is $(-\infty, \infty)$, then the range of function is

(a) $[-2, \infty)$ (b) $(-\infty, \infty)$

(c) $(-2, 1)$ (d) $(-\infty, -2)$

5. In a ΔABC, right-angled at C, if $\tan A = \frac{1}{\sqrt{3}}$, then find the value of $\sin A \cos B + \cos A \sin B$.

(a) 0 (b) 1 (c) 2 (d) 3

6. The value of $\frac{\cos(90° + \theta)\sec(270° + \theta)\sin(180° + \theta)}{\cos(-\theta)\cos(270° - \theta)\tan(180° + \theta)}$ is equal to

(a) $\operatorname{cosec}\theta$ (b) $\sec\theta$

(c) $\sin\theta$ (d) $-\operatorname{cosec}\theta$

7. If $\frac{i^4 + i^9 + i^{16}}{2 - i^8 + i^{10} + i^3} = a + ib$, then (a, b) is

(a) $(1, 2)$ (b) $(-1, 2)$

(c) $(2, 1)$ (d) $(-2, -1)$

8. If $z + \frac{1}{z} = 1 + i$, then find the value of $\left|z^3 + \frac{1}{z^3}\right|$.

(a) $\sqrt{20}$ (b) $\sqrt{24}$ (c) $\sqrt{26}$ (d) $\sqrt{22}$

9. Solve : $5(2x - 7) - 3(2x + 3) \le 0$ and $2x + 19 \le 6x + 51$

(a) $3 \le x \le 9$ (b) $4 \le x \le 12$

(c) $-7 \le x \le 11$ (d) $-8 \le x \le 11$

10. The solution of $6x < \frac{4x - 1}{2}$ is

(a) $x < \frac{-1}{8}$ (b) $\frac{-1}{8} < x < \frac{1}{4}$

(c) $x < \frac{-1}{8}$ and $x > \frac{1}{4}$ (d) $x > \frac{1}{8}$

11. The letters of word ZENITH are written in all possible ways. If all these words are written out as in a dictionary, then find the rank of the word ZENITH.

(a) 614 (b) 616 (c) 714 (d) 716

12. If ${}^nC_3 + {}^nC_4 > {}^{n+1}C_3$, then

(a) $n > 6$ (b) $n > 7$ (c) $n < 6$ (d) $n < 4$

13. A student has to answer 10 questions, choosing at least 4 from each of the parts A and B. If there are 6 questions in part A and 7 in part B, then the number of ways can the student choose 10 questions is

(a) 256 (b) 352 (c) 266 (d) 426

14. The value of $(1+\sqrt{2})^7 + (1-\sqrt{2})^7$ is

(a) 475 (b) 476 (c) 477 (d) 478

15. If $(1 + x - 2x^2)^8 = a_0 + a_1x + a_2x^2 + ... + a_{16}x^{16}$, then the sum of $a_0 + a_2 + a_4 + ... + a_{16}$ is equal to

(a) -2^7 (b) 2^7

(c) 2^8 (d) None of these

16. If 6^{th} term of a G.P. is 2, then the product of first 11 terms of the G.P. is equal to

(a) 512 (b) 1024 (c) 2048 (d) 256

17. The sum of an infinite numbers of a G.P. is 2 and the sum of their cubes is 24. The second term of the G.P. is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) -2 (d) $-\frac{3}{2}$

18. If the slope of the line joining the points (3, 4) and $(-2, a)$ is equal to $-\frac{2}{5}$, then the value of a is equal to

(a) 6 (b) 4 (c) 3 (d) 2

19. A line passing through (3, 5) and perpendicular to the line $3x + y = 3$. Its x intercept is given by

(a) 6 (b) −4 (c) −12 (d) 12

20. A circle passes through the points (2, 3) and (4, 5). If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to

(a) 1 (b) 2 (c) $\sqrt{5}$ (d) $\sqrt{2}$

21. The eccentricity of ellipse $5x^2 + 4y^2 = 1$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{1}{3}$

22. The length of the foot of perpendicular drawn from the point $P(3, 4, 5)$ on y-axis is

(a) 10 units (b) $\sqrt{34}$ units

(c) $\sqrt{113}$ units (d) $5\sqrt{2}$ units

23. The value of $\lim\limits_{x\to 3} \frac{x^5 - 3^5}{x^8 - 3^8}$ is equal to

(a) $\frac{5}{8}$ (b) $\frac{5}{64}$ (c) $\frac{5}{216}$ (d) $\frac{1}{27}$

24. For each $x \in R$, let $[x]$ be the greatest integer less than or equal to x. Then $\lim\limits_{x\to 0^-} \frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to

(a) 0 (b) 1 (c) sin1 (d) −sin1

25. Median of ${}^{2n}C_0, {}^{2n}C_1, {}^{2n}C_2, {}^{2n}C_3, ..., {}^{2n}C_n$ (where n is even) is

(a) ${}^{2n}C_{\frac{n}{2}}$ (b) $\frac{{}^{2n}C_{n+1}}{2}$

(c) $\frac{{}^{2n}C_{n-1}}{2}$ (d) none of these

26. The mean of five observations is 4 and their variance is 5.2. If three of these observations are 2, 4 and 6, then the other two observations are

(a) 3 and 5 (b) 2 and 6 (c) 4 and 4 (d) 1 and 7

27. If A, B and C are three mutually exclusive events, then

(a) $P(A)=\frac{1}{2}, P(B)=\frac{2}{5}, P(C)=\frac{1}{5}$

(b) $P(A) + P(B) + P(C) = 1$

(c) $P(A) + P(B) + P(C) \le 1$

(d) none of these

28. The probability of happening of an event A is 0.2 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B is

(a) 0.4 (b) 0.5 (c) 0.2 (d) 0.9

29. Two letters are chosen from the letters of the word 'EQUATIONS'. The probability that one is vowel and the other is consonant is

(a) $\frac{8}{9}$ (b) $\frac{3}{9}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

30. The relation R defined on the set N of natural number by $x\,R\,y \Leftrightarrow 2x^2 - 3xy + y^2 = 0$ is

(a) symmetric but not reflexive

(b) only symmetric

(c) not symmetric but reflexive

(d) none of the above

31. If a function $f: [2, \infty) \to A$ defined by $f(x) = x^2 - 4x + 5$ is a bijection, then A is equal to

(a) R (b) $[1, \infty)$

(c) $[2, \infty)$ (d) none of these

32. $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right), \frac{\pi}{4} \le x \le \frac{5\pi}{4}$ is equal to

(a) $\frac{\pi}{4} - x$ (b) $x - \frac{\pi}{4}$

(c) $\frac{3\pi}{4} - x$ (d) $x + \frac{\pi}{4}$

33. The value of $\sin\left(\tan^{-1}\frac{1}{3}\right) + \cos\left(\tan^{-1} 2\sqrt{2}\right)$ is

(a) $\frac{14}{15}$ (b) $-\frac{14}{15}$

(c) $\frac{13}{12}$ (d) None of these

34. If $A = \begin{bmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix}$, then find the matrix X such that $3A - 4B + 5X = C$.

(a) $\begin{bmatrix} -1 & 2/5 \\ 6/5 & 19/5 \\ 19/5 & 26/5 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 6/5 & 19/5 \\ 2/5 & 19/5 & 26/5 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2/5 \\ 6/5 & 19/5 \\ 19/5 & 26/5 \end{bmatrix}$ (d) None of these

35. If $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, then which of the following option is correct ?

(a) Both AB and BA exist

(b) Neither AB nor BA exists

(c) AB exists but BA does not exist

(d) AB does not exist but BA exists

36. Find x, if $\begin{bmatrix} 1 & 2 & x \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$ is singular.

(a) 1 (b) 2 (c) 3 (d) 4

37. The value of λ for which the homogeneous system of equations possesses a non-trivial solution $x + \lambda y + 2z = 0$, $3x + 2\lambda y + z = 0$, $2x + 3y - 4z = 0$ is

(a) 0 (b) 15 (c) $\frac{15}{2}$ (d) $-\frac{15}{2}$

38. Let $f(x) = \begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2}, & \text{if } x \neq 2 \\ b, & \text{if } x = 2 \end{cases}$

If $f(x)$ is continuous for all x, then b is equal to

(a) 7 (b) 3 (c) 2 (d) 5

39. If $y = f(x^2 + 2)$ and $f'(3) = 6$, then $\frac{dy}{dx}$ at $x = 1$ is

(a) 25 (b) 12 (c) 5 (d) 15

40. For $f(x) = \begin{cases} x^2 + bx + c, & x < 1 \\ x, & x \geq 1 \end{cases}$.

If $f(x)$ is differentiable at $x = 1$, then $(b - c) =$

(a) -2 (b) 0 (c) 1 (d) 2

41. Find the value of λ if the points $(1, -5)$, $(-4, 5)$, $(\lambda, 7)$ are collinear.

(a) -4 (b) -5 (c) -10 (d) 15

42. Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then $\lim_{t \to 0} \frac{f(t)}{t^2}$ is equal to

(a) 0 (b) -1 (c) 2 (d) 3

43. The edge of a cube is decreasing at the rate of 0.05 cm/sec. Find the rate of decrease of its surface area, when the edge of the cube is 20 cm.

(a) 18 cm^2/sec (b) 8 cm^2/sec

(c) 12 cm^2/sec (d) 15 cm^2/sec

44. For what value of a, $f(x) = -x^3 + 4ax^2 + 2x - 5$ is decreasing $\forall\ x$?

(a) ± 5 (b) 3

(c) 0 (d) Can't say

45. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1 [a > 0]$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a is equal to

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 3

46. A circular plate of radius 5 cm is heated. Due to expansion, its radius increases at the rate of 0.05 cm/sec. The rate at which its area is increasing when the radius is 5.2 cm is

(a) 5.05 π cm^2/sec (b) 5.2 π cm^2/sec

(c) 0.52 π cm^2/sec (d) 27.4 π cm^2/sec

47. Evaluate $\int \frac{e^x(1+x)}{\sin(xe^x)}\,dx$.

(a) $\log\left|\cot\left(\frac{xe^x}{2}\right)\right| + c$ (b) $\log\left|\tan\left(\frac{xe^x}{2}\right)\right| + c$

(c) $\log\left|\sin\left(\frac{xe^{-x}}{2}\right)\right| + c$ (d) $\log\left|\cot\left(\frac{xe^{-x}}{3}\right)\right| + c$

48. If $\int \frac{(x+1)dx}{x + x^2e^x} = A \ln\left(\frac{e^{-x}}{x} + 1\right) + B$, then $A =$

(a) 1 (b) $\frac{1}{2}$ (c) -1 (d) -2

49. If $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx = \frac{k}{4}$, then value of k equals

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) None of these

50. The value of $\int_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}} + 5^{\sqrt{10-x}}}\,dx$ is

(a) 4 (b) 5 (c) 3 (d) 6

51. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} xf(x)dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$ and the x-axis. Then

(a) $R_1 = 2R_2$ (b) $R_1 = 3R_2$

(c) $2R_1 = R_2$ (d) $3R_1 = R_2$

52. Solve the initial value problem

$xe^{y/x} - y + x\frac{dy}{dx} = 0,\ y(e) = 0.$

(a) $y = -x \log(\log|x|);\ x \in (-\infty, -1) \cup (1, \infty)$

(b) $y = \log(x),\ x \in (-\infty, -1) \cup (1, \infty)$

(c) $y = x,\ x \in (-\infty, -1) \cup (1, \infty)$

(d) $y = x + \log x,\ x \in (-\infty, -1) \cup (1, \infty)$

53. Integrating factor of the differential equation $(1+x^2)\frac{dy}{dx}+xy=x$ is

(a) $\frac{x}{1+x^2}$ (b) $\frac{1}{2}\log(1+x^2)$

(c) $\sqrt{1+x^2}$ (d) x

54. The vectors $60\hat{i}+3\hat{j}$, $40\hat{i}-8\hat{j}$ and $p\hat{i}-52\hat{j}$ are collinear, if p =

(a) – 40 (b) 40 (c) 20 (d) 30

55. Let $\vec{u}$, $\vec{v}$, $\vec{w}$ be such that $|\vec{u}|=1, |\vec{v}|=2, |\vec{w}|=3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}$, $\vec{w}$ are perpendicular to each other, then $|\vec{u}-\vec{v}+\vec{w}|$ equals

(a) $\sqrt{14}$ (b) $\sqrt{7}$ (c) 2 (d) 14

56. A line is inclined at 60° with x-axis and 45° with y-axis. Its inclination with z-axis is

(a) 30° (b) 45° (c) 60° (d) 90°

57. Find the shortest distance between the lines whose vector equations are $\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$.

(a) $\frac{1}{\sqrt{59}}$ unit (b) $\sqrt{59}$ units

(c) $\frac{10}{\sqrt{59}}$ units (d) $\frac{10}{\sqrt{5}}$ units

58. The maximum value of $z = 5x + 2y$ subject to $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, $y \ge 0$ is

(a) 10 (b) 0 (c) 6 (d) 12

59. Suppose A and B are such events that $P(A) = 0.3$ $P(A \cup B) = 0.8$. If A and B are independent events, then $P(B)$ is

(a) $\frac{5}{7}$ (b) $\frac{5}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

60. Three cooks X, Y and Z bake a special kind of cake, and with respective probabilities 0·02, 0·03 and 0·05, it fails to rise. In the restaurant where they work, X bakes 50%, Y bakes 30% and Z bakes 20% of cakes. What is the proportion of failures caused by X?

(a) $\frac{9}{29}$ (b) $\frac{10}{29}$ (c) $\frac{19}{29}$ (d) $\frac{28}{29}$

SOLUTIONS

1. (c) : We have, A = {... –4, 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, ...}

B = {... –6, 0, 6, 12,18, 24, 30, 36, ...}

$A \cap B$ = {..., –12, 0, 12, 24, 36, ...}

$\therefore$ $A \cap B = \{x : x = 12n, n \in Z\}$

2. (b)

3. (c) : R = {(4, 3), (8, 2), (12, 1)}

Domain of R = {4, 8, 12}

4. (a) : We have, $f(x) = x^2 - 6x + 7 = (x - 3)^2 - 2$

Let $y = (x - 3)^2 - 2$

$\Rightarrow$ $y + 2 = (x - 3)^2$ $\Rightarrow$ $x-3=\pm\sqrt{y+2}$

$\Rightarrow$ $x=3\pm\sqrt{y+2}$ is defined for $y \ge -2$

$\therefore$ Range is $[-2, \infty)$.

5. (b) : Given, ΔABC is right-angled at C and

$\tan A=\frac{1}{\sqrt{3}} = \tan 30° \Rightarrow \angle A = 30°$

By applying angle sum property in ΔABC,

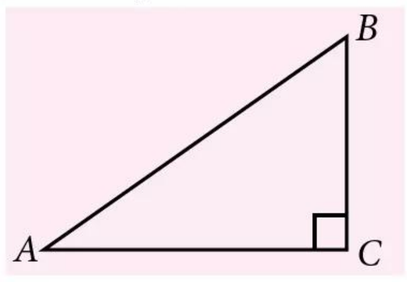
We have $\angle A + \angle B + \angle C = 180°$

$\Rightarrow$ $30° + \angle B + 90° = 180°$

$\Rightarrow$ $\angle B = 180° - 120° = 60°$

$\therefore$ $\sin A \cos B + \cos A \sin B$

$\sin(A + B) = \sin(90°) = 1$



6. (d) : We have, $\frac{\cos(90°+\theta)\sec(270°+\theta)\sin(180°+\theta)}{\cos(-\theta)\cos(270°-\theta)\tan(180°+\theta)}$

$=\frac{-\sin\theta\times\operatorname{cosec}\theta\times(-\sin\theta)}{\cos\theta\times(-\sin\theta)\times\tan\theta}=-\frac{\sin\theta}{\sin^2\theta}=-\operatorname{cosec}\theta$

7. (b) : We have, $\frac{i^4+i^9+i^{16}}{2-i^8+i^{10}+i^3}=a+ib$

or $a+ib=\frac{1+i+1}{2-1+i^2+(-i)}=\frac{2+i}{1-1-i}=\frac{2+i}{-i}$

$\therefore$ $(a, b) \equiv (-1, 2)$

8. (c) : We have, $z+\frac{1}{z}=1+i$; $\left(z+\frac{1}{z}\right)^3=(1+i)^3$

$\Rightarrow$ $z^3+\frac{1}{z^3}+3z\times\frac{1}{z}\left(z+\frac{1}{z}\right)=1+i^3+3\cdot1^2\cdot i+3\cdot1\cdot i^2$

$\Rightarrow$ $z^3+\frac{1}{z^3}+3(1+i)=1-i+3i-3$

$\Rightarrow$ $z^3+\frac{1}{z^3}=-2+2i-3-3i=-5-i$

$\therefore$ $\left|z^3+\frac{1}{z^3}\right|=\sqrt{(-5)^2+(-1)^2}=\sqrt{25+1}=\sqrt{26}$

9. (d): Consider the inequation

$5(2x - 7) - 3(2x + 3) \le 0$

$\Rightarrow \quad 10x - 35 - 6x - 9 \le 0$

$\Rightarrow \quad 4x \le 44 \Rightarrow x \le 11$ ...(i)

Consider the inequation,

$2x + 19 \le 6x + 51$

$\Rightarrow \quad 19 - 51 \le 6x - 2x$

$\Rightarrow \quad -32 \le 4x \Rightarrow -8 \le x \Rightarrow x \ge -8$ ...(ii)

From (i) and (ii), we get $-8 \le x \le 11$

10. (a): Given inequality is, $6x < \dfrac{4x-1}{2}$

$\Rightarrow \quad 12x < 4x - 1 \Rightarrow x < \dfrac{-1}{8}$

11. (b): Writing the letters of word ZENITH alphabetically that is EHINTZ.

Word starting with	Number of words
E	5!
H	5!
I	5!
N	5!
T	5!
ZEH	3!
ZEI	3!
ZENH	2!
ZENIH	1
Total number of words before ZENITH	615

Hence, there are 615 words before ZENITH, the rank of ZENITH is 616.

12. (a): We have, ${}^{n}C_3 + {}^{n}C_4 > {}^{n+1}C_3$

$\Rightarrow \quad {}^{n+1}C_4 > {}^{n+1}C_3 \qquad ({}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1})$

$\Rightarrow \quad \dfrac{{}^{n+1}C_4}{{}^{n+1}C_3} > 1 \Rightarrow \dfrac{n-2}{4} > 1 \Rightarrow n > 6$

13. (c): Required number of ways = Total ways – number of ways choosing at most 3 questions from part A

$= {}^{13}C_{10} - {}^{6}C_3 = 286 - 20 = 266$

14. (d): We know, $(1 + x)^7 + (1 - x)^7$

$= 2[{}^{7}C_0(1)^7 + {}^{7}C_2(1)^5 x^2 + {}^{7}C_4 (1)^3 x^4 + {}^{7}C_6(1)^1 x^6]$

$= 2[1 + 21x^2 + 35x^4 + 7x^6]$

Now, put $x = \sqrt{2}$, we get

$\left(1+\sqrt{2}\right)^7 + \left(1-\sqrt{2}\right)^7 = 2(1+21\times2+35\times4+7\times8)$

$= 2 \times 239 = 478$

15. (b): We have, $(1 + x - 2x^2)^8$

$= a_0 + a_1 x + a_2 x^2 + ... + a_{16} x^{16}$ ...(i)

Put $x = 1$ in (i), we get

$a_0 + a_1 + a_2 + ... + a_{16} = 0$ ...(ii)

Put $x = -1$ in (i), we get

$a_0 - a_1 + a_2 .... + a_{16} = 2^8$ ...(iii)

Adding (ii) and (iii), we get

$2(a_0 + a_2 + ... + a_{16}) = 2^8$

$\Rightarrow \quad a_0 + a_2 + ... + a_{16} = 2^7$

16. (c): Consider the 11 terms of the G.P. as,

$\dfrac{a}{r^5}, \dfrac{a}{r^4}, \dfrac{a}{r^3}, \dfrac{a}{r^2}, \dfrac{a}{r}, a, ar, ar^2, ar^3, ar^4, ar^5$

Now, $T_6 = a = 2$

$\therefore \quad$ Product of 11 terms $= a^{11} = 2^{11} = 2048$

17. (d): Let $a, ar, ar^2, ...$ be the given infinite G.P.

Given, $\dfrac{a}{1-r} = 2, \dfrac{a^3}{1-r^3} = 24$

Eliminating a, we get $24(1 - r^3) = 8(1 - r)^3$

$\Rightarrow \quad 3(1 + r + r^2) = 1 - 2r + r^2$

$\Rightarrow \quad 2r^2 + 5r + 2 = 0$

$\Rightarrow \quad r = -2, -1/2$

But $|r| < 1 \therefore r = -1/2 \Rightarrow a = 3$

$\therefore \quad t_2 = ar = -\dfrac{3}{2}$

18. (a): Two point slope form is $\dfrac{y_2 - y_1}{x_2 - x_1} = m$

$\Rightarrow \quad \dfrac{a-4}{-2-3} = \dfrac{-2}{5} \Rightarrow \dfrac{a-4}{-5} = \dfrac{-2}{5}$

$\Rightarrow \quad a - 4 = 2 \Rightarrow a = 6$

19. (c): Equation of the line $\perp$ to $3x + y - 3 = 0$ is given by $x - 3y + k = 0$

$\therefore k = 12$ as $(3, 5)$ lies on the $\perp$ line.

Now, $x - 3y = -12 \Rightarrow \dfrac{x}{-12} + \dfrac{y}{4} = 1$

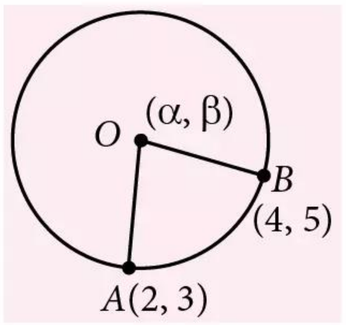
$\Rightarrow \quad x$-intercept is -12.

20. (b): Let radius be r and centre be (α, β).

$\because$ (α, β) lies on $y - 4x + 3 = 0$

$\therefore$ $\beta - 4\alpha + 3 = 0 \Rightarrow \beta = 4\alpha - 3$ ...(i)

$\therefore$ Centre $\equiv (\alpha, 4\alpha - 3)$ ...(ii)



Now, $OA^2 = OB^2$

$\Rightarrow$ $(\alpha - 2)^2 + (\beta - 3)^2$

$= (\alpha - 4)^2 + (\beta - 5)^2$

$\Rightarrow$ $\alpha^2 + 4 - 4\alpha + \beta^2 + 9 - 6\beta$

$= \alpha^2 + 16 - 8\alpha + \beta^2 + 25 - 10\beta$

$\Rightarrow$ $4\alpha + 4\beta = 28 \Rightarrow \alpha + \beta = 7$

$\Rightarrow$ $\alpha + 4\alpha - 3 = 7$ (Using (i))

$\Rightarrow$ $5\alpha = 10 \Rightarrow \alpha = 2$

$\therefore$ Centre $(\alpha, \beta) = (2, 5)$

and radius $= \sqrt{OA} = \sqrt{(2-2)^2 + (5-3)^2} = 2$ units

21. (c): Given, $5x^2 + 4y^2 = 1$

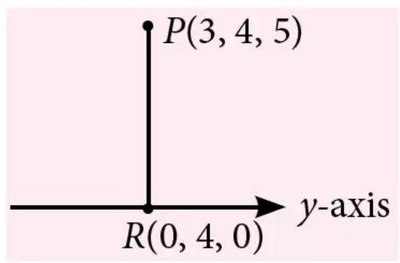
or $\dfrac{x^2}{1/5} + \dfrac{y^2}{1/4} = 1 \Rightarrow a^2 = \dfrac{1}{5}, b^2 = \dfrac{1}{4} \Rightarrow a < b$

Thus the eccentricity of the given ellipse is given by

$$e = \sqrt{\frac{b^2 - a^2}{b^2}} = \sqrt{\frac{\frac{1}{4} - \frac{1}{5}}{\frac{1}{4}}} = \sqrt{\frac{\frac{1}{20}}{\frac{1}{4}}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$$

22. (b): Let $R(0, 4, 0)$ be a point on y-axis.

$\therefore$ Required length,



$$PR = \sqrt{(0-3)^2 + (4-4)^2 + (0-5)^2}$$

$$= \sqrt{9+0+25} = \sqrt{34} \text{ units}$$

23. (c): $\lim\limits_{x \to 3} \dfrac{x^5 - 3^5}{x^8 - 3^8} = \lim\limits_{x \to 3} \dfrac{x^5 - 3^5}{(x-3)} \cdot \left(\dfrac{x-3}{x^8 - 3^8} \right)$

$$= \frac{5 \cdot 3^4}{8 \cdot 3^7} = \frac{5}{8 \cdot 3^3} = \frac{5}{216}$$

24. (d): We have, $\lim\limits_{x \to 0^-} \dfrac{x([x] + |x|) \sin[x]}{|x|}$

$$= \lim_{x \to 0^-} \frac{x(-1-x)\sin(-1)}{-x} \qquad \left[\because \lim_{x \to 0^-} [x] = -1 \text{ and } \lim_{x \to 0^-} |x| = -x \right]$$

$= -\sin 1$

25. (a): ${}^{2n}C_0$, ${}^{2n}C_1$, ${}^{2n}C_2$,...., ${}^{2n}C_n$ is odd number of binomial coefficient (when n is even) and middle binomial coefficient is ${}^{2n}C_{n/2}$ which is required median.

26. (d): Let the unknown observations be a and b.

$$\text{Variance} = \frac{1}{n}\sum x^2 - \bar{x}^2 = 5.2$$

$$\Rightarrow \frac{1}{5}(4+16+36+a^2+b^2) - 16 = 5.2$$

$\Rightarrow$ $a^2 + b^2 = 50$ ...(i)

Also $2 + 4 + 6 + a + b = 20 \Rightarrow a + b = 8$ ...(ii)

It is clear that the two observations are 1 and 7 which satisfies both (i) and (ii).

27. (c): $P(A) + P(B) + P(C) \le 1$ as A, B, C are mutually exclusive. Since A, B, C are not given to be exhaustive, therefore, $P(A) + P(B) + P(C)$ may or may not be equal to 1.

28. (b): $\because$ A and B are mutually exclusive events.

$\therefore$ $P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.3 = 0.5$

Thus, $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - 0.5 = 0.5$

29. (d): The word 'EQUATIONS' has 9 letters, among them 5 are vowels and 4 are consonants.

$\therefore$ Required probability $= \dfrac{{}^5C_1 \times {}^4C_1}{{}^9C_2} = \dfrac{5 \times 4}{9 \times 4} = \dfrac{5}{9}$

30. (c): The relation R on the set of natural number N is defined by $xRy \Leftrightarrow 2x^2 - 3xy + y^2 = 0 \ \forall \ x, y \in N$

Reflexive : Let $x \in N$

$\therefore$ $2x^2 - 3x \cdot x + x^2 = 2x^2 - 3x^2 + x^2 = 0 \Rightarrow xRx$

$\therefore$ R is reflexive.

Symmetric : Let $x, y \in N$, such that $(x, y) \in R$

$\therefore$ $(x, y) \in R \Rightarrow 2x^2 - 3xy + y^2 = 0$

$\Rightarrow$ $2y^2 - 3xy + x^2 \ne 0 \ \forall \ x \ne y \Rightarrow (y, x) \notin R$

$\because$ $(x, y) \in R$ but $(y, x) \notin R$ $\therefore$ R is not symmetric.

31. (b): Since given function, $f(x) = x^2 - 4x + 5$ is bijective function. Therefore, it is one-one and onto.

Let $f(x) = y = x^2 - 4x + 5 \Rightarrow x^2 - 4x + (5 - y) = 0$

$$\Rightarrow x = \frac{4 \pm \sqrt{16 - 4(5-y)}}{2} = \frac{4 \pm \sqrt{-4+4y}}{2}$$

For x to be real, $-4 + 4y \ge 0$ or $y \ge 1$ ...(i)

Since $x \in [2, \infty)$

$\therefore$ $x = \dfrac{4 \pm \sqrt{-4+4y}}{2} = 2$ $\therefore$ For $x = 2, y = 1$ ...(ii)

$\therefore$ $A = [1, \infty)$ [From (i) and (ii)]

32. (b): Given expression

$$= \cos^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right), \frac{\pi}{4}\le x\le\frac{5\pi}{4}$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right), \frac{\pi}{4}\le x\le\frac{5\pi}{4}$$

$$= \cos^{-1}\left(\sin\frac{\pi}{4}\sin x+\cos\frac{\pi}{4}\cos x\right), 0\le x-\frac{\pi}{4}\le\pi$$

$$= \cos^{-1}\left(\cos\left(x-\frac{\pi}{4}\right)\right),\ x-\frac{\pi}{4}\in[0,\pi]$$

$$= x-\frac{\pi}{4}$$

33. (a): $\sin\left(\tan^{-1}\frac{3}{4}\right)+\cos\left(\tan^{-1}\ 2\sqrt{2}\right)$

$$= \sin\left(\sin^{-1}\frac{3}{5}\right)+\cos\left(\cos^{-1}\frac{1}{3}\right)=\frac{3}{5}+\frac{1}{3}=\frac{14}{15}$$

34. (a)

35. (c): Order of matrix $A = 3\times 2$

Order of matrix $B = 2\times 2$

So, matrix AB exists but BA does not exist.

36. (d): For singular matrix, $|A| = 0$

$$\therefore\quad \begin{vmatrix}1 & 2 & x\\ 1 & 1 & 1\\ 2 & 1 & -1\end{vmatrix}=0$$

$\Rightarrow\quad 1(-1-1)-2(-1-2)+x(1-2)=0$

$\Rightarrow\quad -2+6-x=0$

$\Rightarrow\quad x=4$

37. (c): For non-trivial solution, $\Delta = 0$

$$\therefore\quad \begin{vmatrix}1 & \lambda & 2\\ 3 & 2\lambda & 1\\ 2 & 3 & -4\end{vmatrix}=0$$

$\Rightarrow\quad 1(-8\lambda-3)-\lambda(-12-2)+2(9-4\lambda)=0$

$\Rightarrow\quad -8\lambda+14\lambda-8\lambda=-18+3$

$\Rightarrow\quad -2\lambda=-15$

$\Rightarrow\quad \lambda=\frac{15}{2}$

38. (a): $f(x)=\begin{cases}\dfrac{x^3+x^2-16x+20}{(x-2)^2}, & \text{if } x\neq 2\\ b, & \text{if } x=2\end{cases}$

$$\therefore\quad \lim_{x\to 2}f(x)=\lim_{x\to 2}\frac{x^3+x^2-16x+20}{(x-2)^2}$$

$$=\lim_{x\to 2}\frac{(x-2)(x^2+3x-10)}{(x-2)^2}$$

$$=\lim_{x\to 2}\frac{(x-2)(x+5)(x-2)}{(x-2)^2}$$

$$=\lim_{x\to 2}(x+5)=2+5=7$$

$\because\quad f(x)$ is continuous for all x.

$\therefore\quad f(2)=\lim_{x\to 2}f(x)\quad\Rightarrow\quad b=7$

39. (b): $y=f(x^2+2)\Rightarrow\frac{dy}{dx}=f'(x^2+2)\times 2x$

$$\Rightarrow\quad \left(\frac{dy}{dx}\right)_{\text{at } x=1}=f'(3)\times 2=6\times 2=12$$

40. (a): $f(x)=\begin{cases}x^2+bx+c\ , & x<1\\ x\ , & x\ge 1\end{cases}$

$f(x)$ is differentiable at $x = 1$

$\Rightarrow\quad f'(1^-)=f'(1^+)=f'(1)$

And $f(x)$ is continuous at $x = 1$

$\therefore\quad f'(x)=2x+b$

$f'(1)=1\ \Rightarrow\ 2+b=1$

$\Rightarrow\quad b=-1$ and $f(1^-)=f(1^+)$

$\Rightarrow\quad (1)+b+c=1\ \Rightarrow\ b+c=0$

$\Rightarrow\quad b=-c\ \Rightarrow\ c=1$

$\therefore\quad b-c=-1-1=-2$

41. (b): The given points are collinear iff

$$\begin{vmatrix}1 & -5 & 1\\ -4 & 5 & 1\\ \lambda & 7 & 1\end{vmatrix}=0$$

$\Rightarrow\quad 1(-2)+5(-4-\lambda)+1(-28-5\lambda)=0$

$\Rightarrow\quad -50-10\lambda=0\Rightarrow\lambda=-5$

42. (a): $f(t)=\begin{vmatrix}\cos t & t & 1\\ 2\sin t & t & 2t\\ \sin t & t & t\end{vmatrix}$

Expanding along C_2, we get

$f(t) = -t(t\cos t-\sin t)$

Now, $\lim_{t\to 0}\frac{f(t)}{t^2}=\lim_{t\to 0}\frac{-t(t\cos t-\sin t)}{t^2}$

$$= \lim_{t\to 0} -\frac{(t\cos t - \sin t)}{t} = \lim_{t\to 0} -\cos t + \frac{\sin t}{t}$$

$$= -\lim_{t\to 0}\cos t + \lim_{t\to 0}\frac{\sin t}{t} = -1+1 = 0$$

43. (c) : Let s and a be the surface area and edge of the cube respectively at any time t.

$$\Rightarrow \quad s = 6a^2 \Rightarrow \frac{ds}{dt} = 6\times 2a\frac{da}{dt}$$

Given $a = 20$ cm and $\frac{da}{dt} = -0.05$

$$\Rightarrow \frac{ds}{dt} = -6\times 2\times 20\times 0.05 = -12 \text{ cm}^2/\text{sec}$$

Hence the surface area decreases at the rate of 12 cm^2/sec.

44. (d)

45. (a) : Given, $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1,\ a > 0$

$\Rightarrow \quad f'(x) = 6x^2 - 18ax + 12a^2$

Now, $f'(x) = 0 \Rightarrow 6(x^2 - 3ax + 2a^2) = 0$

$\Rightarrow \quad (x - a)(x - 2a) = 0$

$\Rightarrow \quad x = a$ or $x = 2a$

Also, $f''(x) = 12x - 18a$

$\Rightarrow \quad f''(a) = 12a - 18a = -6a < 0$

and $f''(2a) = 24a - 18a = 6a > 0$

Thus, $f(x)$ attains maximum and minimum at a and $2a$ respectively.

$\Rightarrow \quad p = a$ and $q = 2a$

Also $p^2 = q$ (Given)

$\Rightarrow \quad a^2 = 2a$

$\Rightarrow \quad a(a - 2) = 0 \Rightarrow a = 0$ or 2

But $a > 0$ (Given) $\therefore\ a = 2$

46. (c) : We know that area of circle, $A = \pi r^2$

We have, $\frac{dr}{dt} = 0.05$ cm/sec

Now, $\frac{dA}{dt} = 2\pi r\frac{dr}{dt} = 2\pi(5.2)\times 0.05$ [$\because\ r = 5.2$ cm]

$\Rightarrow \quad \frac{dA}{dt} = 0.52\pi \text{ cm}^2/\text{sec}$

47. (b): Let $\text{I} = \int \frac{e^x(1+x)}{\sin(xe^x)}dx$

Put $xe^x = t \Rightarrow e^x(x + 1)\,dx = dt$

$$\therefore\ \text{I} = \int \frac{dt}{\sin t} = \int \text{cosec } t\ dt = \log|\text{cosec } t - \cot t| + c$$

$$= \log\left|\frac{1-\cos t}{\sin t}\right| + c = \log\left|\frac{2\sin^2\frac{t}{2}}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right| + c$$

$$= \log|\tan t/2| + c = \log\left|\tan\left(\frac{xe^x}{2}\right)\right| + c$$

48. (c) : Let $I = \int \frac{(x+1)dx}{x + x^2e^x} = \int \frac{(x+1)dx}{x(1+xe^x)}$

Let $t = 1 + xe^x$

$\Rightarrow \quad dt = (x + 1)e^x dx$

$$\therefore \quad I = \int \frac{dt}{(t-1)t} = \int\left(\frac{1}{t-1} - \frac{1}{t}\right)dt = \ln\left|\frac{t-1}{t}\right| + B$$

$$= \ln\left(\frac{xe^x}{1+xe^x}\right) + B = -\ln\left(\frac{e^{-x}}{x} + 1\right) + B \quad \Rightarrow \quad A = -1.$$

SAMURAI SUDOKU

ANSWER - FEBRUARY 2025

1	9	6	7	2	5	4	3	8				7	4	5	2	6	9	1	3	8
8	4	2	1	3	9	7	6	5				2	1	6	3	5	8	4	7	9
5	3	7	4	6	8	9	1	2				3	8	9	4	7	1	6	2	5
3	8	5	2	4	1	6	9	7				5	3	1	6	9	2	8	4	7
2	1	9	6	5	7	3	8	4				9	7	8	1	3	4	5	6	2
6	7	4	9	8	3	2	5	1				6	2	4	5	8	7	3	9	1
4	5	1	3	7	6	8	2	9	7	6	4	1	5	3	7	2	6	9	8	4
7	6	8	5	9	2	1	4	3	5	9	2	8	6	7	9	4	5	2	1	3
9	2	3	8	1	4	5	7	6	1	8	3	4	9	2	8	1	3	7	5	6
						3	8	7	9	4	5	2	1	6						
						4	1	2	6	3	8	5	7	9						
						6	9	5	2	1	7	3	8	4						
3	4	6	7	9	1	2	5	8	4	7	9	6	3	1	2	7	8	4	5	9
5	8	9	6	4	2	7	3	1	8	2	6	9	4	5	1	6	3	7	8	2
7	1	2	3	8	5	9	6	4	3	5	1	7	2	8	9	5	4	6	1	3
1	5	4	2	6	7	8	9	3				1	7	9	5	3	6	8	2	4
8	2	3	1	5	9	4	7	6				4	8	3	7	1	2	9	6	5
6	9	7	4	3	8	5	1	2				2	5	6	4	8	9	1	3	7
9	6	1	5	2	4	3	8	7				3	6	7	8	9	5	2	4	1
4	3	5	8	7	6	1	2	9				5	9	4	6	2	1	3	7	8
2	7	8	9	1	3	6	4	5				8	1	2	3	4	7	5	9	6

49. (b) : Given, $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx = \frac{k}{4}$ ...(i)

By using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$$\int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \frac{k}{4} \quad \text{...(ii)}$$

On adding (i) and (ii), we get

$$\int_{\pi/6}^{\pi/3} 1\,dx = \frac{k}{2} \Rightarrow \frac{\pi}{6} = \frac{k}{2} \qquad \therefore\ k = \frac{\pi}{3}$$

50. (c) : Let $I = \int_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}dx$ ...(i)

By using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$$\Rightarrow \quad I = \int_2^8 \frac{5^{\sqrt{10-10+x}}}{5^{\sqrt{10-x}}+5^{\sqrt{x}}}dx$$

$$\Rightarrow \quad I = \int_2^8 \frac{5^{\sqrt{x}}}{5^{\sqrt{10-x}}+5^{\sqrt{x}}}dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}dx + \int_2^8 \frac{5^{\sqrt{x}}}{5^{\sqrt{10-x}}+5^{\sqrt{x}}}dx$$

$$\Rightarrow \quad 2I = \int_2^8 \frac{5^{\sqrt{10-x}}+5^{\sqrt{x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}dx = \int_2^8 1\,dx$$

$$\Rightarrow \quad 2I = [x]_2^8 = 6 \Rightarrow I = 3$$

51. (c) : $R_2 = \int_{-1}^{2} f(x)dx$

$$R_1 = \int_{-1}^{2} xf(x)dx = \int_{-1}^{2} (1-x)f(1-x)dx$$

We have, on addition

$$2R_1 = \int_{-1}^{2} f(x)dx = R_2 \qquad [\text{as } f(x) = f(1-x)]$$

52. (a) : We have, $xe^{y/x} - y + x\frac{dy}{dx} = 0$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y - xe^{y/x}}{x}$$

Which is a homogeneous differential equation.

Putting $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = \frac{vx - xe^{v}}{x} = v - e^{v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = -e^{v} \Rightarrow \int -\frac{1}{e^{v}}dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad e^{-v} = \log|x| + c \Rightarrow e^{-y/x} = \log|x| + c$$

It is given that $y(e) = 0$

$$\Rightarrow \quad e^{0} = \log|e| + c \Rightarrow 1 = 1 + c$$

$$\Rightarrow \quad c = 0$$

$$\Rightarrow \quad e^{-y/x} = \log|x|$$

$$\Rightarrow \quad y = -x\log(\log|x|),\ x \in (-\infty, -1) \cup (1, \infty),$$

which is the required solution

53. (c) : We have, $\frac{dy}{dx} + \frac{x}{1+x^2}\cdot y = \frac{x}{1+x^2}$,

which is linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x.

$$\therefore \quad \text{I.F.} = e^{\int P\cdot dx}$$

$$e^{\int \frac{x}{1+x^2}\cdot dx} = e^{\frac{1}{2}\cdot\int \frac{2x}{1+x^2}\cdot dx} = e^{\frac{1}{2}\log(1+x^2)} = e^{\log(1+x^2)^{1/2}}$$

So, I.F. $= \sqrt{1+x^2}$

54. (a) : Let $\vec{a} = 60\,\hat{i} + 3\hat{j}$, $\vec{b} = 40\,\hat{i} - 8\hat{j}$, $\vec{c} = p\hat{i} - 52\hat{j}$

Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = -20\hat{i} - 11\hat{j}$

$$\overrightarrow{AC} = \vec{c} - \vec{a} = (p-60)\hat{i} - 55\hat{j}$$

Given vectors are collinear,

$$\therefore\ \overrightarrow{AB} = k\overrightarrow{AC} \Rightarrow -20\hat{i} - 11\hat{j} = k(p-60)\hat{i} - 55k\hat{j}$$

$$\Rightarrow \quad -55k = -11 \text{ and } k(p-60) = -20 \Rightarrow k = \frac{1}{5}$$

Hence, $\frac{p-60}{5} = -20 \Rightarrow p - 60 = -100 \Rightarrow p = -40$

55. (a) : Given $\frac{\vec{v}\cdot\vec{u}}{|\vec{u}|}=\frac{\vec{u}\cdot\vec{w}}{|\vec{u}|}$ and $\vec{v}\cdot\vec{w}=0$

Also $|\vec{u}|=1, |\vec{v}|=2, |\vec{w}|=3$

Now $|\vec{u}-\vec{v}+\vec{w}|^2=|\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2$
$-2\vec{u}\cdot\vec{v}-2\vec{v}\cdot\vec{w}+2\vec{u}\cdot\vec{w}$

$= 1 + 4 + 9 + 0 = 14$

56. (c) : $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow \quad \cos^2 60°+\cos^2 45°+\cos^2\gamma=1$

$\therefore \quad \cos^2\gamma=1-\frac{1}{4}-\frac{1}{2}=\frac{1}{4} \Rightarrow \cos\gamma=\frac{1}{2}, \gamma=60°.$

57. (c) : $\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$

$\therefore \ \vec{a}_1=\hat{i}+\hat{j}, \vec{b}_1=2\hat{i}-\hat{j}+\hat{k}$

Also, $\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

$\therefore \ \vec{a}_2=2\hat{i}+\hat{j}-\hat{k}, \vec{b}_2=3\hat{i}-5\hat{j}+2\hat{k}$

So, $\vec{a}_2-\vec{a}_1=\hat{i}-\hat{k}$

And, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-1&1\\3&-5&2\end{vmatrix}=3\hat{i}-\hat{j}-7\hat{k}$

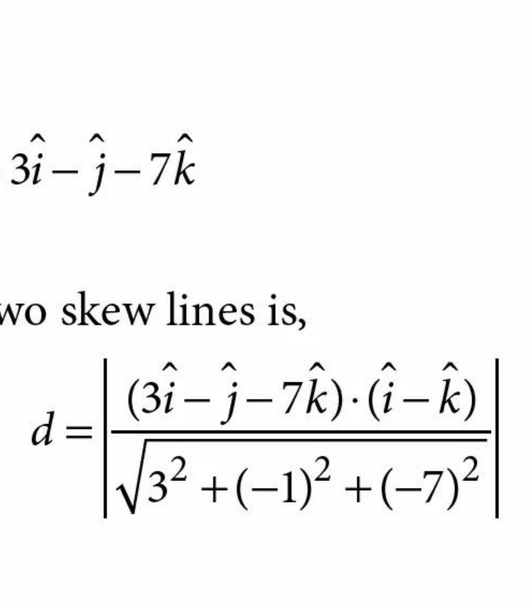
Shortest distance between two skew lines is,

$$d=\left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right| \Rightarrow d=\left|\frac{(3\hat{i}-\hat{j}-7\hat{k})\cdot(\hat{i}-\hat{k})}{\sqrt{3^2+(-1)^2+(-7)^2}}\right|$$

$$\Rightarrow \quad d=\left|\frac{3+7}{\sqrt{59}}\right|=\frac{10}{\sqrt{59}} \text{ units.}$$

58. (a) : Converting inequations into equations and drawing the corresponding lines.

$3x + 5y = 15, 5x + 2y = 10$



i.e. $\frac{x}{5}+\frac{y}{3}=1, \ \frac{x}{2}+\frac{y}{5}=1$

As $x \geq 0, y \geq 0$ solution lies in first quadrant

B is the point of intersection of the lines $3x+5y=15$ and $5x+2y=10$

i.e. $B=\left(\frac{20}{19}, \frac{45}{19}\right)$

We have points $O(0, 0), A(2, 0), B\left(\frac{20}{19}, \frac{45}{19}\right)$ and $C(0, 3)$

Now, $z = 5x + 2y$

$\therefore \quad z(O) = 5(0) + 2(0) = 0$

$z(A) = 5(2) + 2(0) = 10$

$z(B)=5\left(\frac{20}{19}\right)+2\left(\frac{45}{19}\right)=10$

$z(C) = 5(0) + 2(3) = 6$

$\therefore$ z has maximum value 10 at two points $A(2, 0)$ and $B\left(\frac{20}{19}, \frac{45}{19}\right)$.

59. (a) : Given that A and B are independent events, then

$P(A \cap B) = P(A) \times P(B) = 0.3\times P(B)$ ...(i)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad 0.8 = 0.3 + P(B) - 0.3 \times P(B)$ [From (i)]

$\Rightarrow \quad 0.5 = 0.7\ P(B) \Rightarrow P(B) = \frac{5}{7}$

60. (b) : Let F be the event that cake fails to rise.

A be the event cook X bakes cake

B be the event cook Y bakes cake

and C be the event cook Z bakes cake.

$P(A) = 0.5, P(B) = 0.3$ and $P(C) = 0.2$

Also, $P(F|A) = 0.02$

$P(F|B) = 0.03$ and $P(F|C) = 0.05$

P(Proportion of failures caused by X) $= P(A|F)$

$$=\frac{P(F|A)P(A)}{P(F|A)P(A)+P(F|B)\cdot P(B)+P(F|C)P(C)}$$

$$=\frac{0.02\times0.5}{0.02\times0.5+0.03\times0.3+0.05\times0.2}$$

$$=\frac{0.01}{0.01+0.009+0.01}=\frac{0.01}{0.029}=\frac{10}{29}$$

Recipe for Success

"Coming together is a beginning; Keeping together is progress; Working together is success."

–Henry Ford

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

1. Evaluate the following indefinite integral:

$$\int \frac{dx}{(x+1)^5\sqrt{x^2+2x}}$$ ***(Priya Malhotra, Gujarat)***

Ans. Let $I = \int \frac{dx}{(x+1)^5\sqrt{x^2+2x}} = \int \frac{dx}{(x+1)^5\sqrt{(x+1)^2-1}}$

$$= \int \frac{dx}{(x+1)^6\sqrt{1-\frac{1}{(x+1)^2}}} = \int \frac{-t^4 dt}{\sqrt{1-t^2}} \left[\text{Putting } \frac{1}{x+1} = t\right]$$

Here, t^4 is a polynomial of degree 4.

$$\therefore \quad I = \int \frac{-t^4 dt}{\sqrt{1-t^2}} = (At^3 + Bt^2 + Ct + D)\sqrt{1-t^2} + \lambda \int \frac{dt}{\sqrt{1-t^2}} \quad \ldots(i)$$

Differentiating both sides and multiplying by $\sqrt{1-t^2}$, we get

$-t^4 = (3At^2 + 2Bt + C)(1 - t^2) - t(At^3 + Bt^2 + Ct + D) + \lambda$

Comparing the coefficients, we have

$-4A = -1, 3B = 0, 3A - 2C = 0, 2B - D = 0$ and $C + \lambda = 0$

$\Rightarrow \quad A = 1/4, B = 0, C = 3/8, D = 0$ and $\lambda = -3/8$

From (i), $I = \left(\frac{1}{4}t^3 + \frac{3}{8}t\right)\sqrt{1-t^2} - \frac{3}{8}\int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{1}{8}(2t^3 + 3t)\sqrt{1-t^2} - \frac{3}{8}\sin^{-1} t + C$, where $t = \frac{1}{x+1}$.

2. Find the range of the given function:

$$y = \frac{1}{2 + \sin 3x + \cos 3x}.$$ ***(Sidhartha, Bihar)***

Ans. We have, $y = \frac{1}{2 + \sin 3x + \cos 3x}$

$$= \frac{1}{2 + \sqrt{2}\sin(3x + \pi/4)}$$

The value of $\sqrt{2}\sin(3x + \pi/4)$ lies between $-\sqrt{2}$ and $\sqrt{2}$ for all $x \in R$; the denominator never becomes zero. Hence, the domain is $x \in R$.

Now, $\sqrt{2}\sin(3x + \pi/4) = \frac{1}{y} - 2$

For values of x belonging to the domain ($x \in R$), we have

$-\sqrt{2} \le \sqrt{2}\sin(3x + \pi/4) \le \sqrt{2}$

i.e., $-\sqrt{2} \le \frac{1}{y} - 2 \le \sqrt{2} \Rightarrow 2 - \sqrt{2} \le \frac{1}{y} \le 2 + \sqrt{2}$

$$\Rightarrow \quad \frac{1}{2+\sqrt{2}} \le y \le \frac{1}{2-\sqrt{2}}$$

Hence, the range is $\left[\frac{1}{2+\sqrt{2}}, \frac{1}{2-\sqrt{2}}\right]$.

3. Let $\bar{b}z + b\bar{z} = c$ is the equation of a straight line. If z_1 and z_2 be mirror image of each other in this line, then prove that $\bar{b}z_2 + b\bar{z}_1 = c$. ***(Rituraj, Madhya Pradesh)***

Ans. If z_1 and z_2 are mirror image of each other in the line

$$\bar{b}z + b\bar{z} = c \quad \ldots(i)$$

Then the mid-point of z_1, z_2 must lie on (i)

and the line joining z_1, z_2 must be perpendicular to (i)

$$\therefore \quad \bar{b}\left(\frac{z_1+z_2}{2}\right) + b\overline{\left(\frac{z_1+z_2}{2}\right)} = c$$

$$\Rightarrow \quad \bar{b}(z_1 + z_2) + b(\bar{z}_1 + \bar{z}_2) = 2c$$

$$\Rightarrow \quad (\bar{b}z_1 + b\bar{z}_2) + (\bar{b}z_2 + b\bar{z}_1) = 2c \quad \ldots(ii)$$

Putting $z = x + iy$ in equation (i), it reduces to

$$\bar{b}(x + iy) + b(x - iy) - c = 0$$

i.e., $(\bar{b} + b)x + i(\bar{b} - b)y - c = 0$

whose slope $= \frac{(b+\bar{b})}{i(b-\bar{b})}$

Equation of the line joining z_1, z_2 is given by

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$$

i.e., $z\bar{a} - \bar{z}a + d = 0$, where $a = z_1 - z_2$

and $d = z_1\bar{z}_2 - \bar{z}_1 z_2$

Putting $z = x + iy$, the above equation reduces to

$$\bar{a}(x + iy) - a(x - iy) + d = 0$$

i.e., $(\bar{a} - a)x + i(\bar{a} + a)y + d = 0$

whose slope $= \frac{(a-\bar{a})}{i(a+\bar{a})}$, Now, $\frac{(b+\bar{b})}{i(b-\bar{b})} \times \frac{(a-\bar{a})}{i(a+\bar{a})} = -1$

$$\Rightarrow \quad (b+\bar{b})(a-\bar{a}) = (b-\bar{b})(a+\bar{a}) \Rightarrow \bar{b}a = b\bar{a}$$

$$\Rightarrow \quad \bar{b}(z_1 - z_2) = b(\bar{z}_1 - \bar{z}_2)$$

$$\Rightarrow \quad \bar{b}z_1 + b\bar{z}_2 = \bar{b}z_2 + b\bar{z}_1 \quad \ldots(iii)$$

From equations (ii) and (iii), we have $\bar{b}z_2 + b\bar{z}_1 = c$, which is the desired result. ◈◈

Are you ready for JEE 2025?

Series 1

Top Weightage Topics for JEE

In this article we will discuss about some of the most important topics of mathematics in respect to JEE Main and Advanced. It is based on the analysis of previous years' questions of JEE Main and Advanced.

SETS

A well-defined collection of distinct objects is known as set.

Representation of Sets

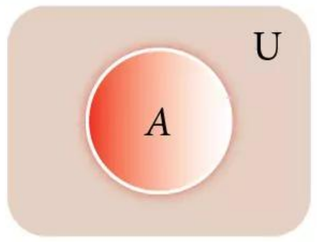
Set Builder Form or Rule Method:

$A = \{x : x \text{ is a prime number}\}$

Roster or Tabular Form :

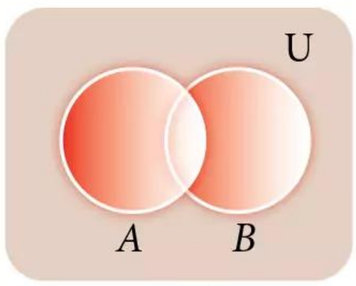
$A = \{2, 3, 5, 7\}$.

Compliment of a Set



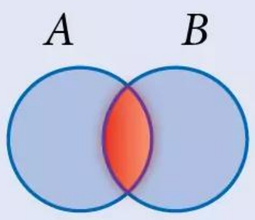
A' or $A^C = U - A$

Union of two sets



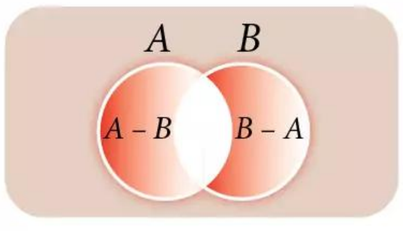
$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Intersection of two sets



$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Symmetric difference of A and B



$A\Delta B = (A - B) \cup (B - A)$; $x \in A \cup B$ but $x \in A \cap B$

Demorgan's law

- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

Application of Sets

If A, B and C are finite sets and U be the finite universal set, then

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B)$, if A, B are disjoint.
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
- $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

RELATIONS

If A and B are two non-empty sets, then a relation R from A to B is a subset of $A \times B$. Also, total number of relations from A to B is 2^{mn}.

Types of Relations

Reflexive Relation

A relation R on a set A is reflexive, if $(a, a) \in R$ $\forall\ a \in A$.

Transitive Relation

A relation R on a set A is transitive, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R\ \forall\ a_1, a_2, a_3 \in A$.

Symmetric Relation

A relation R on a set A is symmetric, if $(a_1, a_2) \in R \Rightarrow (a_2, a_1) \in R\ \forall\ a_1, a_2 \in A$.

Equivalence Relation

A relation R is an equivalence relation if R is reflexive, symmetric and transitive.

Inverse Relation

A relation R from A to B has inverse R^{-1} from B to A, $R^{-1} = \{(b, a) : (a, b) \in R\}$.

Composition of Relations

A relation R from A to B, relation S from B to C, then SoR is a relation from A to C, $(a, c) \in$ SoR $\Leftrightarrow \exists\ b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$.

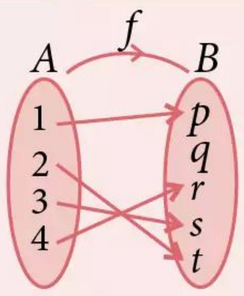
FUNCTIONS

A relation 'f' from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

Types of Functions

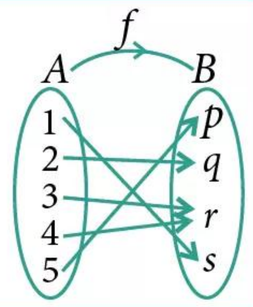
One-one (Injective) Function

A function $f : A \to B$ is one-one, if no two elements of A have same image in B.



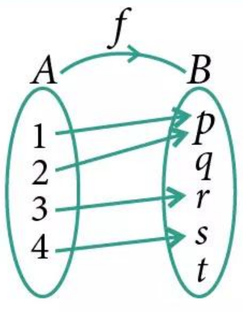
Onto (Surjective) Function

A function $f : A \to B$ is onto, if all the elements of B have atleast one pre-image in A.



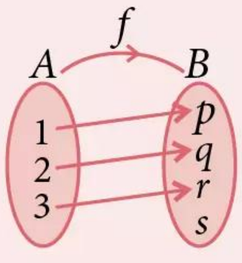
Many-one Function

A function $f : A \to B$ is many-one, if two or more than two elements of A have the same images in B.



Into Function

A function $f : A \to B$ is into, if there exists atleast single element in B having no pre-image in A.



Bijective Function

A function which is both one-one & onto.

- $n(A) = n(B)$
- Range = Co-domain

Invertible Functions

Let $f : A \to B$ be a bijection. Then the inverse function f^{-1} exists from B to A. *i.e.*, $f^{-1} : B \to A$

$\therefore \quad f^{-1}(y) = x \Leftrightarrow y = f(x)$

Some Real Functions

- **Signum Function**

$$y = f(x) = \text{sgn}(x) = \begin{cases} -1, & \text{if} \quad x < 0 \\ 0\ , & \text{if} \quad x = 0 \\ 1\ , & \text{if} \quad x > 0 \end{cases}$$

- **Greatest Integer Function**

$y = f(x) = [x] = n \quad \text{if} \quad n \le x < n + 1$

- **Least Integer Function**

$y = f(x) = (x) = n + 1 \quad \text{if} \quad n < x \le n + 1$

- **Fractional Part Function**

$y = f(x) = \{x\} = f \quad \text{if} \quad x = n + f, n \in I \text{ and } 0 \le f < 1.$

- **Exponential Function**

$y = f(x) = a^x, a > 0$

- **Logarithmic Function**

$y = f(x) = \log_a x, a > 0, a \ne 1 \text{ and } x > 0$

Number of Functions

Let $f : A \to B, |A| = m, |B| = n.$

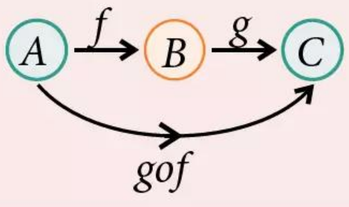
- The number of functions from A to B is n^m.
- The number of one-one functions from A to $B = \begin{cases} {}^nP_m, \text{if } n \ge m \\ 0, \quad \text{if } n < m \end{cases}$
- The number of onto functions from A to $B = \begin{cases} \sum_{r=1}^{n} (-1)^{n-r}\ {}^nC_r\, r^m, & \text{if} \quad n \le m \\ n!, & \text{if} \quad n = m \\ 0, & \text{if} \quad n > m \end{cases}$
- The number of bijections from A to $B = \begin{cases} n!, & m = n \\ 0, & n > m \text{ or } m > n \end{cases}$

Periodic Functions

$y = f(x) = f(T + x), T > 0 \ \forall \ x \in \text{Dom}\ (f)$

Composition of Functions

Let $f : A \to B$ and $g : B \to C$ be two functions, then $gof : A \to C$ is known as composition of functions.



Properties of Composition of Functions

- $fog \ne gof$
- $fo(goh) = (fog)oh$ (Associative property)
- Composition of two bijections is also a bijection.

COMPLEX NUMBERS

Combination of both real number and imaginary number are called complex number.

Definition

A number of the form $a + ib$, where $a, b \in R$ and $i = \sqrt{-1}$ is called a complex number, denoted by 'z'.

$z = a + ib$

Real part; Imaginary part

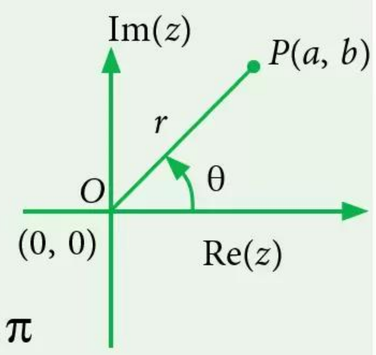
Square Root of Complex Number

$$\sqrt{a+ib} = \begin{cases} \pm\left(\sqrt{\frac{|z|+a}{2}} + i\sqrt{\frac{|z|-a}{2}}\right), & \text{for } b > 0 \\ \pm\left(\sqrt{\frac{|z|+a}{2}} - i\sqrt{\frac{|z|-a}{2}}\right), & \text{for } b < 0 \end{cases}$$

Modulus, Conjugate & Argument

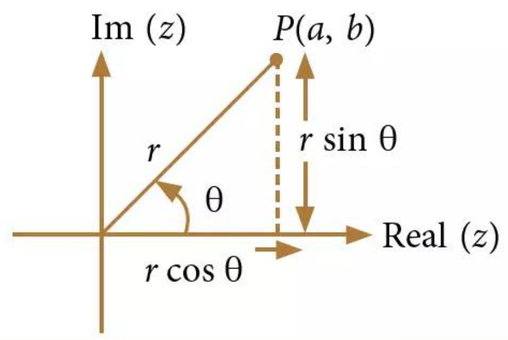
- Modulus of $z = a + ib$ is $r = |z| = \sqrt{a^2 + b^2}$
- Conjugate of z is $\overline{z} = a - ib$
- Argument of $z = a + ib$ is

$$\theta = \tan^{-1}\left(\frac{\text{Im}(z)}{\text{Re}(z)}\right), \quad -\pi < 0 \le \pi$$



Polar Representation

$z = r[\cos(2n\pi + \theta) + i \sin(2n\pi + \theta)], n \in I$



Cube Roots of Unity

Let $x = (1)^{1/3} \Rightarrow x^3 - 1 = 0 \Rightarrow x = 1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2} \Rightarrow x = 1, \omega, \omega^2$

- $\omega^3 = 1$
- $1 + \omega + \omega^2 = 0$
- $1 + \omega^n + \omega^{2n} = \begin{cases} 0, & \text{if } n \text{ is not a multiple of 3} \\ 3, & \text{if } n \text{ is a multiple of 3} \end{cases}$

De-Moivre's Theorem

$z = r(\text{cis}\,\theta), z^n = r^n(\text{cis}\, n\theta)$

Also, n^{th} roots of unity is given by $z^n = 1$ and

$z = \text{cis}\left(\frac{2k\pi}{n}\right), k = 0, 1, 2, \ldots, n-1.$

Euler's Form

$z = re^{i\theta}$, $\overline{z} = re^{-i\theta}$, where $-\pi < \theta \le \pi$, θ is the principal argument.

QUADRATIC EQUATIONS

An equation of the form $ax^2 + bx + c$ is called a quadratic equation, where $a, b, c \in R$ or C and $a \neq 0$.

Nature of the Roots of Quadratic Equation

$\Delta = b^2 - 4ac$ is the discriminant of $ax^2 + bx + c = 0$, a, b, c are real.

- If $\Delta > 0$, then the roots are real and distinct.
- If $\Delta = 0$, then the roots are real and equal.
- Let $\Delta < 0$, then the roots are complex conjugates and solution is given as $x = \frac{-b \pm \sqrt{4ac - b^2}}{2a} i$.
- Let a, b, c be integers and Δ be a square of an integer. Then the roots of $ax^2 + bx + c = 0$ are rational numbers.

Common Roots of Quadratic Equation

Condition of one common root

Let $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ have a common root α.

$$\therefore \alpha = \frac{b_1c_2 - b_2c_1}{a_2c_1 - a_1c_2} = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}, \alpha \neq 0$$

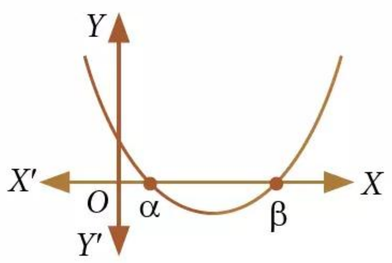
Condition for both the common roots

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Quadratic Expressions and their Graphs

Case-I : For $a > 0$ and $D > 0$

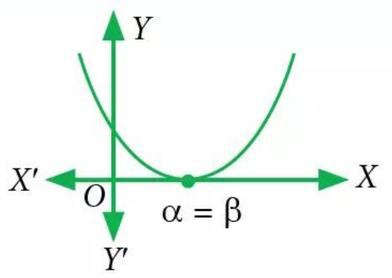
For $\alpha < x < \beta$, $y < 0$



Case-II : For $a > 0$ and $D = 0$

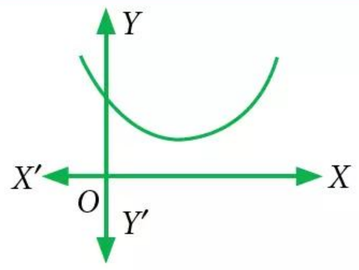
$y = a(x - \alpha)^2$

and $y \geq 0$, for $x \in R$.



Case-III : For $a > 0$ and $D < 0$

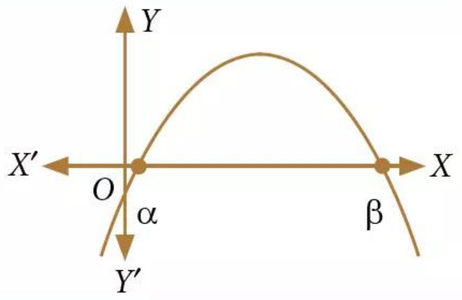
In this case, imaginary roots appears & $y > 0$ for $x \in R$.



Case-IV : For $a < 0$ and $D > 0$

$y > 0$, if $\alpha < x < \beta$

$y < 0$, if $x < \alpha$ or $x > \beta$



LOGARITHM

Logarithm is the power to which a number must be raised in order to get some other number.

Properties of Logarithm

- $\log_b 1 = 0$
- $\log_b b^x = x$
- $\log_b a = \log_c a \log_b c$
- $\log_b (xy) = \log_b x + \log_b y$
- $\log_{b^\beta} a = \frac{1}{\beta}\log_b a$
- $a^{\log_c b} = b^{\log_c a}$ ($a, b, c > 0$ and $c \neq 1$)
- $\log_b b = 1$
- $b^{\log_b x} = x$
- $\log_b a = \frac{1}{\log_a b}$
- $\log_b (x/y) = \log_b x - \log_b y$
- $\log_{b^\beta} a^\alpha = \frac{\alpha}{\beta}\log_b a$
- $\log_2 \log_2 \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{...\sqrt{2}}}}}} = -n$, where n is the number of square roots.

ARITHMETIC PROGRESSION (A.P.)

A.P. is a sequence in which each term is obtained by adding a fixed number to the preceding term except the first term.

Sum of n Terms of an A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d] \text{ or } S_n = \frac{n}{2}[a + l], \text{ where } l = \text{last term}$$

General Term (or n^{th} term) of an A.P.

$$a_n = n^{th} \text{ term} = a + (n-1)d$$

n^{th} Term from last of an A.P.

$a'_n = l - (n-1)d$, where n = number of terms

Arithmetic Mean A.M.

- For two numbers a and b, A.M. is $\frac{a+b}{2}$.
- $A_r = a + r\left(\frac{b-a}{n+1}\right), \forall\ r = 1, 2, \ldots, n$

where $A_1, A_2, \ldots, A_n$ are n arithmetic means inserted between two numbers a and b.

GEOMETRIC PROGRESSION (G.P.)

G.P. is a sequence in which each term is obtained by multiplying the previous term by a non-zero constant.

General Term (or n^{th} term) of a G.P.

$a_n = ar^{n-1}$

n^{th} Term from the Last of a G.P.

$a'_n = \frac{l}{r^{n-1}}$

Sum of 'n' Term of a G.P.

$S_n = \frac{a(1-r^n)}{1-r}, r<1$ or $S_n = \frac{a(r^n-1)}{r-1}, r>1$

or $S_n = na,\ r = 1$

Sum of Infinite Terms of a G.P.

$S_\infty = \frac{a}{1-r}; |r|<1$

Geometric Mean (G.M.)

- For two numbers a and b, G.M. is $\sqrt{ab}$
- $G_k = a\left(\frac{b}{a}\right)^{\frac{k}{n+1}}, \forall\ r = 1, 2, \ldots, n$

Where $G_1, G_2, \ldots, G_n$ are n geometric means inserted between two numbers a and b.

Relation between A.M. and G.M.

If $a_1, a_2, a_3, \ldots, a_n$ are 'n' positive numbers, then A.M. $\geq$ G.M.

i.e., $\frac{a_1 + a_2 + \ldots + a_n}{n} \geq (a_1\ a_2\ \ldots\ a_n)^{\frac{1}{n}}$; Equality holds if $a_1 = a_2 = \ldots = a_n$

Sum of n Terms of Special Series

- Sum of n natural numbers $= \sum n = \frac{n(n+1)}{2}$
- Sum of squares of n natural numbers $= \sum n^2 = \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of n natural numbers $= \sum n^3 = \frac{n^2(n+1)^2}{4} = \left(\sum n\right)^2$

PERMUTATIONS AND COMBINATIONS

Permutation is an arrangement of items, where the order matters, while a combination is a selection of items, where the order does not matter.

Factorial Notation

Product of first n natural numbers is denoted by $n!$

- $n! = n(n-1)(n-2)\ldots 3\cdot 2\cdot 1$

Permutations

Arranging r objects out of n different things,

- when repetition is not allowed $= {}^nP_r = \frac{n!}{(n-r)!}$, where $0 \leq r \leq n$
- when repetition is allowed $= n^r$

Combinations

Selecting r objects out of n different things

$$= {}^nC_r = \frac{n!}{r!(n-r)!},\ 0 \le r \le n$$

Properties

- ${}^nP_r = {}^nC_r \cdot r!,\ 0 \le r \le n$
- For $0 \le r \le n$, ${}^nC_r = {}^nC_{n-r}$
- For $1 \le r \le n$, ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_a = {}^nC_b \Rightarrow a = b$ or $n = a + b$

Distributions into Groups

Distribution of n distinct things into r groups $G_1, G_2, ..., G_r$ containing $P_1, P_2, ..., P_r$ elements respectively,

- Groups are distinct : $\frac{n!}{P_1!\,P_2!\,...\,P_r!}r!$
- Groups are identical : $\frac{n!}{P_1!\,P_2!\,...\,P_r!}$

BINOMIAL THEOREM

Binomial theorem is a mathematical formula that expands the powers of a binomial expression.

Binomial Theorem for any Positive Integer *n*

$$(x+y)^n = {}^nC_0\,x^n y^0 + {}^nC_1 x^{n-1}y^1 + {}^nC_2 x^{n-2}y^2 + ... + {}^nC_n\,x^0 y^n$$

i.e., $(x+y)^n = \sum_{r=0}^{n} {}^nC_r\ x^{n-r}b^r$,

where ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$ are known as binomial coefficients.

- $(x-y)^n = \sum_{r=0}^{n} {}^nC_r(-1)^r\,x^{n-r}\,y^r$
- $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$
- $(1-x)^n = \sum_{r=0}^{n} (-1)^r\ {}^nC_r\ x^r$

Properties of Binomial Coefficients

- $C_0 + C_1 + C_2 + ... + C_n = 2^n$
- $C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ... = 2^{n-1}$
- $C_0 - C_1 + C_2 - ... + (-1)^n\,C_n = 0$
- ${C_0}^2 + {C_1}^2 + {C_2}^2 + ... + {C_n}^2 = {}^{2n}C_n$
- $C_0\,C_r + C_1\,C_{r+1} + C_2\,C_{r+2} + ... + C_{n-r}\,C_n = {}^{2n}C_{n-r}$
- $\sum_{r=0}^{n} r\ {}^nC_r = n2^{n-1}$
- $\sum_{r=0}^{n} r\,(r-1)\ {}^nC_r = n(n-1)\,2^{n-2}$
- $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$
- $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + ... + \frac{(-1)^n C_n}{n+1} = \frac{1}{n+1}$

General Terms

- $T_{r+1} = {}^nC_r\,x^{n-r}\,a^r$
- T_{r+1} from the end

  $= T_{n-r+1}$ from the beginning

Middle Terms

Number of terms $= n + 1$

If n is even, then $T_{\frac{n}{2}+1}$ is the middle term.

If n is odd, then $T_{\frac{n+1}{2}}, T_{\frac{n+3}{2}}$ are two middle terms.

Multinomial Theorem

$$(x_1 + x_2 + ... + x_r)^n = \sum \frac{n!}{p_1!\,p_2!\,...\,p_r!}\,x_1^{p_1}x_2^{p_2}...x_r^{p_r},\ \text{where } p_1 + p_2 + p_3 ... + p_r = n$$

MATRICES

A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix.

Order of a Matrix

A matrix having m rows and n columns is called a matrix of order $m \times n$.

Transpose of a Matrix

Transpose of a matrix is obtained by interchanging rows and columns of the matrix.

If $A = [a_{ij}]_{m \times n}$, then A' or $A^T = [a_{ji}]_{n \times m}$

Properties

- $(A')' = A$
- $(ABC)' = C'\ B'\ A'$
- $(AB)' = B'A'$
- $(kA)' = kA'$
- $(A \pm B)' = A' \pm B'$

DETERMINANTS

A compact form showing a set of numbers arranged in same number of rows and columns is called determinants.

Minors and Cofactors

- For any matrix $A = [a_{ij}]_{n \times n}$, if we delete (remove) the row and column of the element a_{ij}, then the value of determinant thus obtained is called the minor of a_{ij} and it is denoted by M_{ij}.
- The minor M_{ij} multiplied by $(-1)^{i+j}$ is called the cofactor of the element a_{ij} and denoted by C_{ij} or A_{ij}.
  $\therefore\ C_{ij} = (-1)^{i+j} M_{ij}$

Singular and Non-Singular Matrices

A square matrix A of order n is said to be

- Singular if $|A| = 0$
- Non-singular if $|A| \neq 0$

Solution of System of Linear Equations

Let $AX = B$ be the given system of equations :

- If $|A| \neq 0$, the system is consistent and has a unique solution.
- If $|A| = 0$ and $(\text{adj } A)B \neq O$, then the system is inconsistent and hence it has no solution.
- If $|A| = 0$ and $(\text{adj } A)B = O$, then the system is consistent and hence it has infinitely many solutions.

Adjoint of a Matrix

Let $B = [A_{ij}]$ be the matrix of cofactors of matrix $A = [a_{ij}]$. Then the transpose of B is called the adjoint of matrix A.

Properties

If A is non-singular matrix of order n, then

- $A(\text{adj } A) = (\text{adj } A)\ A = |A|\ I_n$
- $|A \text{ adj } A| = |A|^n$
- $\text{adj } (AB) = (\text{adj } B)\ .\ (\text{adj } A)$
- $|\text{adj } A| = |A|^{n-1}$
- $\text{adj } (\text{adj } A) = |A|^{n-2}\ A$
- $|\text{adj } (\text{adj } A)| = |A|^{(n-1)^2}$

Inverse of a Matrix

If A and B are two square matrices such that $AB = BA = I$, then B is the inverse matrix of A and A is the inverse of B. It is denoted by A^{-1}.

$$i.e.\ A^{-1} = \frac{adj\ A}{|A|},\ |A| \neq 0$$

Properties

- Inverse of a square matrix is unique, if it exists.
- $(A^{-1})^{-1} = A$
- $(kA)^{-1} = A^{-1}/k$
- $(AB)^{-1} = B^{-1}\ A^{-1}$
- $(A^T)^{-1} = (A^{-1})^T$
- $(ABC)^{-1} = C^{-1}\ B^{-1}\ A^{-1}$

Challenging PROBLEMS For JEE

ON

Limits, Continuity and Differentiability

SINGLE OPTION CORRECT TYPE

1. If the function $f(x)=\begin{cases}k_1(x-\pi)^2-1, & x\le\pi\\ k_2\cos x, & x>\pi\end{cases}$ is twice differentiable, then the ordered pair (k_1, k_2) is equal to

(a) $(1, 0)$ (b) $\left(\frac{1}{2},1\right)$ (c) $(1, 1)$ (d) $\left(\frac{1}{2},-1\right)$

2. Let $x=2$ be a root of the equation $x^2+px+q=0$ and

$$f(x)=\begin{cases}\dfrac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4}, & x\neq 2p\\ 0, & x=2p\end{cases}$$

Then $\lim\limits_{x\to 2p^+}[f(x)]$, where $[\cdot]$ denotes greatest integer function, is

(a) -1 (b) 0 (c) 2 (d) 1

3. The value of

$$\lim_{x\to 0}\left((\cos x)^{\frac{1}{\sin^2 x}}+\frac{\sin 2x+2\tan^{-1}3x+3x^2}{\ln(1+3x+\sin^2 x)+xe^x}\right)$$ is

(a) $\sqrt{e}+\frac{3}{2}$ (b) $\frac{1}{\sqrt{e}}+\frac{3}{2}$

(c) $\sqrt{e}+2$ (d) $\frac{1}{\sqrt{e}}+2$

4. Let $f(x) = (x^2 - 3x + 2)|(x^3 - 6x^2 + 11x - 6)| + \left|\sin\left(x+\frac{\pi}{4}\right)\right|$. Number of points at which the function $f(x)$ is non-differentiable in $[0, 2\pi]$, is

(a) 5 (b) 4

(c) 3 (d) 2

5. If $\lim\limits_{x\to 0}\left(\left[\frac{\sin^{-1}x}{x}\right]+\left[\frac{2^2\sin^{-1}2x}{x}\right]+\left[\frac{3^2\sin^{-1}3x}{x}\right]+....+\left[\frac{n^2\sin^{-1}nx}{x}\right]\right)=100.$

Then the value of 'n' is ($[k]$ denotes the greatest integer less than or equal to 'k')

(a) 2 (b) 3 (c) 4 (d) 5

6. If $\lim\limits_{x\to\infty}\frac{4^x}{(4-\sqrt{3}+2\sin\theta)^{x+2}}$ exists and is equal to $P(P\neq 0)$ where $\theta\in\left(0,\frac{\pi}{2}\right)$, then the value of $\left(\frac{P+\cos\theta}{P}\right)$ is

(a) 7 (b) 8 (c) 9 (d) 10

7. Let $a_1=1$ and $a_n=n(a_{n-1}+1)$ for $n=2, 3, ...$ define $P_n=\left(1+\frac{1}{a_1}\right)\left(1+\frac{1}{a_2}\right)....\left(1+\frac{1}{a_n}\right)$. Then $\lim\limits_{n\to\infty}P_n$ is

(a) $1+e$ (b) e (c) 1 (d) ∞

8. If $g(x)=\lim\limits_{t\to 0}\tan\left(\frac{e^{2tx}-2tx-1}{2t^2}\right)$ and

$L=\lim\limits_{x\to 0}\frac{g^2(x)-x^4}{x^8}$, then which of the following option(s) is/are true?

(a) $\{L\}=\frac{2}{3}$ (b) $[L]=1$

(c) $\lim\limits_{x\to 0}\frac{g(Lx)}{x^2}=\frac{4}{9}$ (d) $\lim\limits_{x\to 0}\frac{g(Lx)}{x^2}=\frac{25}{9}$

(where { } and [ ] are fractional part function and Greatest integral function respectively)

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9. The value of $\lim\limits_{x\to 0^+}(\ln(\sin^3 x)-\ln(x^4+ex^3))=$

(a) 0 (b) 1

(c) -1 (d) does not exist

10. If $f(2010)=\pi/2$ and $f(x)=f(2x)\ \forall\ x\in R$ and $f(x)$ is continuous for $x\in$ R, then the value of

$$\lim_{x\to 0}\frac{\cos(f(x))-\sqrt[3]{\cos f(x)}}{\sin^2 x}=$$

(a) 1/4 (b) 1

(c) 0 (d) does not exist

MORE THAN ONE OPTIONS CORRECT TYPE

11. Let $f(x)=1+e^{\ln(\ln x)}.\ln(k^2+25)$ and $g(x)=\dfrac{1}{|x|-1}$.

If $\lim\limits_{x\to 1}(f(x))^{g(x)}=k(2\sin^2\alpha+3\cos\beta+5), k>0$

and $\alpha, \beta\in R$, then which of the following options are correct?

(a) The value of 'k' is 5

(b) The value of $\dfrac{\sin^{10}\alpha+\cos^5\beta}{\sin^2\alpha+\cos\beta}$ is equal to 1

(c) $\cos^2\beta+\sin^4\alpha=2$

(d) $\sin^2\alpha>\cos\beta$

12. The possible values of 'k' for which

$$\lim_{x\to\infty}\frac{2x^3-(\tan^{-1}x)^3}{\frac{8}{\pi}x^3\cot^{-1}|kx|+k^2x^6\sin\frac{1}{x^3}-3kx^3}=\frac{1}{2}$$ are

(a) 0 (b) -1 (c) 2 (d) 4

13. Consider the function $f(x)=x^3-9x^2+15x+6$ for $1\le x\le 6$ and

$$g(x)=\begin{cases}\min\{f(t) \text{ for } 1\le t\le x, 1\le x\le 6\\ x-18 \quad \text{for } x>6\end{cases},$$

then which of the following statement(s) is/are correct?

(a) $g(x)$ is continuous at $x=6$

(b) $g(x)$ is differentiable at $x=2$

(c) $g(x)$ is differentiable at $x=5$

(d) $g(x)$ is monotonic in (1, 5)

14. Let $f(x)=\begin{cases}\left[\dfrac{2(\tan^{-1}x-x)}{x^2}\right] & ;\ x<0\\ (x^2-4)|x^2-2x| & ;\ x\ge 0\end{cases}$; then which of the following option(s) is/are correct? ([·] is G. I. F)

(a) Number of points at which $f(x)$ is discontinuous in R is not finite.

(b) Number of points at which $f(x)$ is discontinuous in R is 0.

(c) Number of points at which $f(x)$ is not differentiable in $(-1, \infty)$ is 1.

(d) Number of points at which $f(x)$ is not differentiable in R is 1.

15. Let $f(x)=x^2-2|x|$ and

$$g(x)=\begin{cases}\min f(t), & -2\le t\le x & -2\le x<0\\ \max f(t), & 0\le t\le x & 0\le x\le 2,\\ f(x) & & x>2\end{cases}$$

then $g(x)$ is not differentiable at

(a) $x=0$ (b) $x=-1$ (c) $x=2$ (d) $x=1$

NUMERICAL / INTEGER ANSWER TYPE

16. Let $\lim\limits_{x\to\infty}\dfrac{1+cn^2}{(2n+3+2\sin n)^2}=\dfrac{1}{2}$. If $c\le\alpha\le\beta$ where α and β are the roots of the equation $x^2-2px+p^2-1=0$, then the minimum integral value of 'p' is _____.

17. $\lim\limits_{x\to\infty}\dfrac{d}{dx}\left(\dfrac{8x^2+bx+c}{4x+f}\right)=2m,$

then $\lim\limits_{x\to m^+}2^{-2^{\overline{(1-x)}}}=$ _____.

18. If $L=\lim\limits_{x\to 0}\dfrac{1}{x^5}\left(\ln\left(\dfrac{1+x}{1-x}\right)-2\tan x\right)$, then $\dfrac{1}{L}$ is equal to _____.

SOLUTIONS

1. (b): $f(x)=\begin{cases}k_1(x-\pi)^2-1 & ;\ x\le\pi\\ k_2\cos x & ;\ x>\pi\end{cases}$ ...(i)

$$f'(x)=\begin{cases}2k_1(x-\pi) & ;\ x\le\pi\\ -k_2\sin x & ;\ x>\pi\end{cases}$$ ...(ii)

and $f''(x)=\begin{cases}2k_1 & ;\ x\le\pi\\ -k_2\cos x & ;\ x>\pi\end{cases}$

Since, $f(x)$ is twice differentiable at $x=\pi$ then from equation (i)

$$\lim_{x\to\pi^-}f(x)=\lim_{x\to\pi^+}f(x)\Rightarrow -1=-k_2\Rightarrow k_2=1$$

From equation (ii)

$$\lim_{x\to\pi^+} f''(x) = \lim_{x\to\pi^-} f''(x) \Rightarrow k_2 = 2k_1 \Rightarrow k_1 = \frac{1}{2}$$

2. (b) : $x = 2$ is a root of $x^2 + px + q = 0$

$\Rightarrow \; q + 4 = -2p$

$$\text{Now, } \lim_{x\to 2p^+} \left\{ \frac{1-\cos(x^2-4px+q^2+8q+16)}{(x^2-4px+q^2+8q+16)^2} \times \frac{(x^2-4px+q^2+8q+16)^2}{(x-2p)^4} \right\}$$

$$= \frac{1}{2} \lim_{x\to 2p^+} \frac{(x^2-4px+4p^2)^2}{(x-2p)^4} = \frac{1}{2} \times 1 = \frac{1}{2}$$

$$\therefore \lim_{x\to 2p^+} [f(x)] = 0$$

3. (d) : $e^{\lim\limits_{x\to 0} \frac{(\cos x - 1)}{\sin^2 x}}$

$$+ \lim_{x\to 0} \frac{\frac{\sin 2x}{2x}\cdot 2x + 6x \cdot \frac{\tan^{-1} 3x}{3x} + 3x^2}{\frac{\ln(1+3x+\sin^2 x)}{3x+\sin^2 x}\cdot(3x+\sin^2 x) + xe^x} = \frac{1}{\sqrt{e}} + 2$$

4. (c) : $f(x) = (x^2 - 3x + 2)|(x-1)(x-2)(x-3)|$

$$+ \left|\sin\left(x + \frac{\pi}{4}\right)\right|$$

Not differentiable at $x = 3, \frac{3\pi}{4}, \frac{7\pi}{4}$

5. (c) : In vicinity of $x = 0$, $|\sin^{-1} x| \geq |x|$

$\Rightarrow \; \dfrac{\sin^{-1} x}{x} \geq 1$

$$l = 1^3 + 2^3 + 3^3 + \ldots + n^3 \Rightarrow \frac{n^2(n+1)^2}{4} = 100 \Rightarrow n = 4$$

6. (c) : $P = \lim\limits_{x\to\infty} \left(\dfrac{4}{4-\sqrt{3}+2\sin\theta}\right)^x \dfrac{1}{(4-\sqrt{3}+2\sin\theta)^2}$

$$\text{If } \theta \in \left(0, \frac{\pi}{3}\right) \Rightarrow \sin\theta \in \left(0, \frac{\sqrt{3}}{2}\right) \Rightarrow P \text{ does not exist.}$$

$$\text{If } \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right) \Rightarrow \sin\theta \in \left(\frac{\sqrt{3}}{2}, 1\right) \Rightarrow P = 0$$

For existence of limit, $\theta = \dfrac{\pi}{3} \Rightarrow P = \dfrac{1}{4^2} = \dfrac{1}{16}$

$$\therefore \left(\frac{P+\cos\theta}{P}\right)_{\theta=\frac{\pi}{3}} = 1 + 8 = 9$$

7. (b) : $P_n = \dfrac{1+a_1}{a_1} \times \dfrac{1+a_2}{a_2} \times \ldots\ldots \dfrac{1+a_n}{a_n}$

$$= \frac{a_2}{2\times a_1} \times \frac{a_3}{3\times a_2} \times \ldots\ldots \times \frac{a_n}{n\times a_{n-1}} \times \frac{a_{n-1}}{(n+1)a_n}$$

$$P_n = \frac{a_{n+1}}{\lfloor n+1} = \frac{(n+1)(a_n+1)}{(n+1)\lfloor n}$$

$$= \frac{1}{\lfloor n} + \frac{a_n}{\lfloor n} = \frac{1}{\lfloor n} + \frac{n(1+a_{n-1})}{n\lfloor n-1}$$

$$= \frac{1}{\lfloor n} + \frac{1}{\lfloor n-1} + \ldots\ldots\ldots + \frac{(1+a_1)}{\lfloor 1}$$

$$= 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \ldots\ldots + \frac{1}{\lfloor n}$$

$$\lim_{n\to\infty} P_n = e$$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

	1	6						3				2						5	9	
2			5				4						3				8			2
7				2												5				6
	4				8										9				2	
		3				8								2				7		
			9				6						5				4			
				7				1				8				9				
	9				4			7				4			7				6	
6						9	3			8			1	7						8
									5		9									
								6				9								
									3		8									
3						4	6			3			9	1						7
	6				4			9				3			1				2	
				8				5				6				8				
			7				1						6				8			
		6				2								3				6		
	4				5										4				1	
8				7												7				2
9			8				3						5				9			8
	7	1						4				7						3	4	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

8. (a) : $g(x)=\lim_{t\to 0}\tan\left(\frac{e^{2tx}-2tx-1}{2t^2}\right)=\tan x^2$

$$L=\lim_{x\to 0}\frac{\tan^2 x^2-x^4}{x^8}=\frac{(\tan x^2-x^2)}{x^6}\frac{(\tan x^2+x^2)}{x^2}$$

$$L=\frac{2}{3}$$

9. (c) : $\lim_{x\to 0^+}\ln\left(\frac{\sin^3 x}{x^4+ex^3}\right)=\lim_{x\to 0^+}\ln\left(\frac{\sin^3 x}{x^3}\cdot\frac{1}{x+e}\right)$

$$=\lim_{x\to 0^+}3\ln\left(\frac{\sin x}{x}\right)-\ln(x+e)=-1$$

10. (c) : $f(x)=f(2x)\ \forall\ x\Rightarrow f(x)=$ constant, $f(x)=\frac{\pi}{2}$

$$\therefore\ \lim_{x\to 0}\frac{\cos(\pi/2)-\sqrt[3]{\cos\pi/2}}{\sin^2 x}=0$$

11. (a,b,c) : $\lim_{x\to 1}(1+\ln x\cdot\ln(k^2+25))^{\frac{1}{|x|-1}}$

$$=e^{\lim_{x\to 1}\frac{\ln x\cdot\ln(k^2+25)}{x-1}}$$

$$=k^2+25=k(2\sin^2\alpha+3\cos\beta+5)$$

$$\Rightarrow\ 2\sin^2\alpha+3\cos\beta+5=k+\frac{25}{k}$$

LHS ≤ 10, RHS $\ge 10\ \Rightarrow\ \sin^2\alpha=1, \cos\beta=1$

12. (a,b,d) :

$$L=\lim_{x\to\infty}\frac{2x^3-(\tan^{-1}x)^3}{\frac{8}{\pi}x^3\cot^{-1}|kx|+k^2x^6\sin\frac{1}{x^3}-3kx^3}=\frac{1}{2}$$

$$\Rightarrow\ L=\lim_{x\to\infty}\frac{2-\left(\frac{\tan^{-1}x}{x}\right)^3}{\frac{8}{\pi}\cot^{-1}(kx)+\frac{k^2\sin\frac{1}{x^3}}{\frac{1}{x^3}}-3k}=\frac{1}{2}$$

If $k=0\ \Rightarrow\ L=\frac{2-0}{\frac{8}{\pi}\cdot\frac{\pi}{2}}=\frac{1}{2}$

If $k\ne 0, L=\frac{2}{0+k^2-3k}=\frac{1}{2}\ \Rightarrow\ k=4,-1.$

13. (b,c,d) : $f'(x)=3x^2-18x+15=3(x-5)(x-1)$

f is increasing in (5, 6), decreasing in (1, 5)

$$\therefore\ g(x)=\begin{cases}x^3-9x^2+15x+6, & 1\le x<5\\ -19, & 5\le x\le 6\\ x-18, & x>6\end{cases}$$

14. (c,d) : $0<\frac{2(\tan^{-1}x-x)}{x^2}<1$ as $\tan^{-1}x<\frac{x^2}{2}+x.$

$\forall\ x<0$

as $f(x)=\frac{x^2}{2}+x-\tan^{-1}x$ and

$$f'(x)=x+1-\frac{1}{1+x^2}=0\ \text{ at } x=0$$

and $f''(0)>0\Rightarrow f(x)\ge f(0)$

$$\Rightarrow\ f(x)=\begin{cases}0; & x<0\\ (x^2-4)|x^2-2x|; & x\ge 0\end{cases}$$

$\Rightarrow$ $f(x)$ is continuous $\forall\ x\in R$ but not differentiable at $x=0$.

15. (a,c) : $g(x)=\begin{cases}f(x) & ;-2<x<-1\\ f(-1) & ;-1\le x<0\\ f(0) & ;0\le x\le 2\\ f(x) & ;x>2\end{cases}$

16. (3) : $\lim_{x\to\infty}\frac{1+cn^2}{(2n+3+2\sin n)^2}=\frac{1}{2}\ \Rightarrow c=2$

Both roots lies in [2, ∞)

$D\ge 0, \frac{-b}{2a}\ge 2$ and $f(2)\ge 0\ \Rightarrow\ p\ge 3$

17. (1) : $\lim_{x\to\infty}\frac{d}{dx}\left(\frac{ax^2+bx+c}{ex+f}\right)$

$$=\lim_{x\to\infty}\frac{(ex+f)(2ax+b)-(ax^2+bx+c)e}{(ex+f)^2}$$

$$=\lim_{x\to\infty}\frac{\left(e+\frac{f}{x}\right)\left(2a+\frac{b}{x}\right)-\left(a+\frac{b}{x}+\frac{c}{x^2}\right)e}{\left(e+\frac{f}{x}\right)^2}=\frac{2ae-ae}{e^2}=\frac{a}{e}$$

$$\therefore\ \lim_{x\to\infty}\frac{d}{dx}\left(\frac{8x^2+bx+c}{4x+f}\right)=\frac{8}{4}=2\ \Rightarrow\ m=1$$

$$\lim_{x\to 1^+}2^{-2^{\frac{1}{1-x}}}=1 \qquad \left[\frac{1}{1-x}\to-\infty\right]$$

18. (7.5) : Use series expansion, $L=\frac{2}{5}-2\times\frac{2}{15}=\frac{2}{15}$

$$\therefore\ \frac{1}{L}=7.5$$

QUANTITATIVE APTITUDE

For Various Competitive Exams

1. A shopkeeper wants to sell a certain quantity (in kg) of grains. He sells half the quantity and an additional 3 kg of these grains to the first customer. Then, he sells half of the remaining quantity and an additional 3 kg of these grains to the second customer. Finally, when the shopkeeper sells half of the remaining quantity and an additional 3 kg of these grains to the third customer, there are no grains left. The initial quantity (in kg) of grains is

(a) 42 (b) 18

(c) 36 (d) 50

2. A bus starts at 9 a.m. and follows a fixed route every day. One day, it travelled at a constant speed of 60 km per hour and reached its destination 3.5 hours later than its scheduled arrival time. Next day, it travelled two-thirds of its route in one-third of its total scheduled travel time and the remaining part of the route at 40 km per hour to reach just on time. The scheduled arrival time of the bus is

(a) 10:30 p.m. (b) 7:00 p.m.

(c) 7:30 p.m. (d) 9:00 p.m.

3. A jar contains a mixture of 175 ml water and 700 ml alcohol. Gopal takes out 10% of the mixture and substitutes it by water of the same amount. The process is repeated once again. Now, the percentage of water in the mixture is

(a) 25.4 (b) 20.5

(c) 30.3 (d) 35.2

4. Let both the series $a_1, a_2, a_3, \ldots$ and $b_1, b_2, b_3, \ldots$ be the arithmetic progression such that the common difference of both the series are prime numbers. If $a_5 = b_9$; $a_{19} = b_{19}$ and $b_2 = 0$, then a_{11} equals

(a) 79 (b) 83

(c) 86 (d) 84

5. Pipes A and C are inlet pipes while pipe B is an outlet pipe of a tank. Pipe B empties the full tank in one hour less than the time taken by pipe A to fill the empty tank. When pipes A, B and C are turned on together, the empty tank is filled in two hours. If pipes B and C are turned on together when the tank is empty and pipe B is turned off after one hour, then pipe C takes another one hour and 15 minutes to fill the remaining tank. If pipe A can fill the empty tank in less than five hours, then the time taken (in minutes) by pipe C to fill the empty tank is

(a) 75 (b) 60 (c) 90 (d) 120

6. In an election, there were four candidates and 80% of the registered voters casted their votes. One of the candidates received 30% of the casted votes while the other three candidates received the remaining casted votes in the proportion 1 : 2 : 3. If the winner of the election received 2512 votes more than the candidate with the second highest votes, then the number of registered votes are

(a) 60288 (b) 62800 (c) 50240 (d) 40192

7. Aman buys 110 units of Article A and 120 units of Article B, Article A being 20% less costly than Article B, per units. He sells 10 units of Article A at 10% profit and 20 units of Article B at 20% profit. Mixing the remaining Article B and Article A, Aman sells the mixture at ₹ 308.32 per unit and makes an oveall profit of 64%. Then, cost price for Article A (in ₹ per units) is

(a) 160 (b) 124 (c) 150 (d) 180

8. Nitu has an initial capital of ₹ 20,000. Out of this, she invests ₹ 8,000 at 5.5% in bank A, ₹ 5,000 at 5.6% in bank B and the remaining amount at x% in bank C, each rate being simple interest per annum. Her combined annual interest income from these

investments is equal to 5% of the initial capital. If she had invested her entire initial capital in bank *C*, then her annual interest income (in ₹) would have been

(a) 900 (b) 700 (c) 1000 (d) 800

9. The average score of 40 students in a Mathematics test is 50. If the highest and lowest scores were excluded, the average score of the class would decrease by 1. If the difference of these 2 scores is 60, find the highest score.

(a) 84 (b) 95 (c) 99 (d) 115

10. An alloy of copper and aluminium has 40% copper. An another alloy of copper and zinc has copper and zinc in the ratio 2 : 7. These two alloys are mixed in such a way that in the overall alloy, there is more aluminium than zinc and copper constitutes x% of this alloy. What is the range of values x can take?

(a) $30\% \le x \le 40\%$ (b) $32.5\% \le x \le 42\%$

(c) $33.33\% \le x \le 40\%$ (d) $32.25\% \le x \le 40\%$

11. Consider a square *ABCD*. *EFGH* is another square obtained by joining the midpoints of the sides of the square *ABCD* where *E*, *F*, *G* and *H* are the midpoints of *AB*, *BC*, *CD* and *DA* respectively. Lakshman and Kanika start from points *B* and *D* respectively at speeds l km/h and k km/h respectively and travel towards each other along the sides of the square *ABCD*. Jagdeesh starts from point *E* and travels along the square *EFGH* in the anti-clockwise direction at j km/h. Lakshman and Kanika meet for the second time at *H*, where Jagdeesh also meets them for the first time. If $l : k : j$ is $1 : 3 : 5\sqrt{2}$, then the distance travelled by Jagdeesh is

(a) $7.5 \times \sqrt{2}$ times the side of the square *ABCD*

(b) $7.5 \times \sqrt{2}$ times the side of the square *EFGH*

(c) 7.5 times the side of the square *ABCD*

(d) 7.5 times the side of the square *EFGH*

12. A shopkeeper marked up the price of article *P* by 20% and gave discount of d%, then he gets a loss of $(d - 11)$%. If cost price of article *Q* is ₹ 200 and gets a profit of $1.5d$%, then find the selling price of the article Q.

(a) ₹ 335 (b) ₹ 305 (c) ₹ 300 (d) ₹ 345

13. Atul age is $(3p + 2q)$ years and his only son's age is p years while his only daughter's age is q years. Atul's son is three years elder than her sister. If Atul's wife is five years younger than her husband who is 34 years old, then find average age of the family (in years)

(a) 14.5 (b) 19.0 (c) 16.5 (d) 22.5

14. All the rearrangements of the word "DEMAND" are written without including any word that has two D's appearing together. If these are arranged alphabetically, what would be the rank of DEMAND?

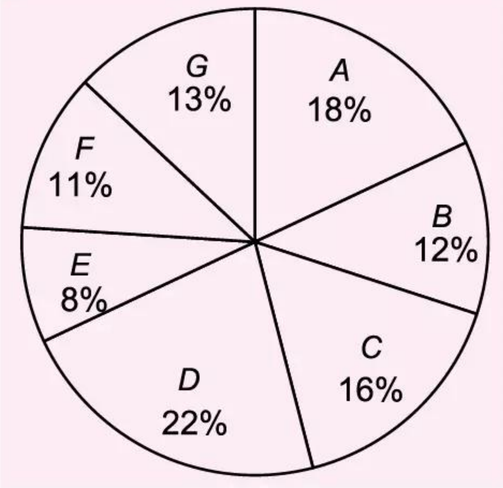
(a) 36 (b) 74 (c) 42 (d) 86

15. Find the remainder when $1821 \times 1823 \times 1827$ is divided by 12.

(a) 9 (b) 12 (c) 3 (d) 6

Directions for questions 16 to 20: Answer the questions on the basis of the information given below:

The below pie chart shows percentage of 4800 students of 7 different schools in 10^{th} class.



The below table shows ratio of boys and girls among these students.

School	Boys : Girls
A	7 : 9
B	5 : 4
C	5 : 7
D	6 : 5
E	3 : 5
F	7 : 4
G	7 : 5

16. What is the ratio of boys in school F to girls in school *E*?

(a) 3 : 4 (b) 5 : 7 (c) 4 : 7 (d) 7 : 5

17. What is the ratio of girls in school D to boys in school C?

(a) 1 : 1 (b) 5 : 7

(c) 3 : 2 (d) 7 : 5

18. What is the percentage of total number of students studying in school B as compared to the total number of students studying in school E?

(a) 150% (b) $66\frac{2}{3}\%$

(c) $33\frac{1}{3}\%$ (d) 120%

19. What is the ratio of number of girls studying in school D and E?

(a) 1 : 4 (b) 1 : 2 (c) 2 : 1 (d) 4 : 1

20. What is the difference in total number of students of school A and E?

(a) 360 (b) 560 (c) 520 (d) 480

SOLUTIONS

1. (a) : Let the inital quantity of grains be A kg.

Quantity sold to first customer $= \left(\frac{A}{2}+3\right)$ kg

Remaining quantity $= A-\left(\frac{A}{2}+3\right)=\left(\frac{A}{2}-3\right)$ kg

Quantity sold to second customer $= \frac{1}{2}\left(\frac{A}{2}-3\right)+3$

$=\left(\frac{A}{4}+\frac{3}{2}\right)$ kg

After selling to the first and second customer, the remaining quantity

$$=\left(\frac{A}{2}-3\right)-\left(\frac{A}{4}+\frac{3}{2}\right)=\left(\frac{A}{4}-\frac{9}{2}\right)\text{kg}$$

Quantity sold to the third customer $= \frac{1}{2}\left(\frac{A}{4}-\frac{9}{2}\right)+3$

$$=\left(\frac{A}{8}+\frac{3}{4}\right)\text{ kg}$$

After selling to the first, second and third customer, the remaining quantity

$$=\left(\frac{A}{4}-\frac{9}{2}\right)-\left(\frac{A}{8}+\frac{3}{4}\right)=\left(\frac{A}{8}-\frac{21}{4}\right)\text{ kg}$$

Given, $\frac{A}{8}-\frac{21}{4}=0 \Rightarrow A=42$ kg

Hence, the initial quantity of grains is 42 kg.

2. (c) : Let the scheduled time be t hours.

From the first statement,

Total distance $= 60 \times (t + 3.5)$

$[\because$ Distance = speed × time$]$

From the second statement,

Time taken to cover two-thirds of route $= \frac{t}{3}$

Remaining part of the route $= 1-\frac{2}{3}=\frac{1}{3}$

Time taken to cover one-third of the route

$$=\frac{1}{3}\times\frac{\text{total distance}}{40}=\frac{1}{3}\times\frac{(t+3.5)}{40}\times 60=\frac{t+3.5}{2}$$

Since, the bus reaches on time, the total time taken is equal to the scheduled time.

$$\frac{t}{3}+\frac{(t+3.5)}{2}=t \Rightarrow \frac{2t+3t+10.5}{6}=t$$

$\Rightarrow t = 10.5$ hour

Scheduled arrival time = 9 a.m. + 10.5 hour = 7:30 p.m.

3. (d) : Initial quantity of alcohol in the mixture = 700 ml

Initial quantity of water in the mixture = 175 ml

Total mixture = 700 + 175 = 875 ml

Final quantity of alcohol in the mixture

$$=700\left(1-\frac{10}{100}\right)^2=567\text{ml}$$

Final quantity of water in the mixture = 875 – 567 = 308 ml

Hence, the percentage of water in the mixture

$$=\frac{308}{875}\times 100=35.2\%$$

4. (a) : Let the common difference of arithmetic progression A and B be d_1 and d_2 respectively, where

$A = a_1, a_2, a_3, \ldots$

$B = b_1, b_2, b_3, \ldots$

Given, $b_2 = 0 \Rightarrow b_1 + (2 - 1)d_2 = 0$

$[\because a_n = a + (n - 1)d]$

$\Rightarrow b_1 = -d_2$ ...(i)

Also, $a_5 = b_9$

$\Rightarrow a_1 + 4d_1 = b_1 + 8d_2$

$\Rightarrow a_1 + 4d_1 = 7d_2$ ... (ii) [Using(i)]

and $a_{19} = b_{19}$

$\Rightarrow a_1 + 18d_1 = b_1 + 18d_2$

$\Rightarrow a_1 + 18d_1 = 17d_2$ ... (iii)

Subtracting equation (iii) from equation (ii), we get

$$a_1 + 18d_1 - (a_1 + 4d_1) = 17d_2 - 7d_2$$

$$\Rightarrow \quad 14d_1 = 10d_2 \quad \Rightarrow \quad \frac{d_1}{d_2} = \frac{10}{14} = \frac{5}{7}$$

Since, d_1 and d_2 are prime numbers, therefore, $d_2 = 7$, $d_1 = 5$

$\therefore \quad a_1 + 4 \times 5 = 7 \times 7$ (Using (ii))

$\Rightarrow \quad a_1 = 49 - 20 = 29$

Hence, $a_{11} = a_1 + 10d_1 = 29 + 10 \times 5 = 29 + 50 = 79$

5. (c) : Let the time taken by pipe C and A to fill the empty tank be y and x hours respectively.

Time taken by pipe B to empty the full tank = $(x - 1)$ hours

When pipes A, B and C are turned together, the empty tank is filled in 2 hours.

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{(x-1)} + \frac{1}{y} = \frac{1}{2} \qquad \ldots (1)$$

Pipes B and C turned on together when the tank is empty and pipe B is turned off after one hour.

$\therefore$ Remaining capacity of tank after 1 hour

$$= 1 + \frac{1}{x-1} - \frac{1}{y}$$

Pipe C takes another 1 hour and 15 minutes to fill the remaining tank.

$$\frac{1.25}{y} = \left(1 + \frac{1}{(x-1)} - \frac{1}{y}\right) \quad \Rightarrow \quad \frac{1.25}{y} + \frac{1}{y} = 1 + \frac{1}{x-1}$$

$$\Rightarrow \quad \frac{2.25}{y} = 1 + \frac{1}{x-1} \quad \Rightarrow \quad \frac{9}{4y} - \frac{1}{(x-1)} = 1 \qquad \ldots (2)$$

Solving equation (1) and (2), we get

$$\frac{1}{x} - \left(\frac{9}{4y} - 1\right) + \frac{1}{y} = \frac{1}{2} \quad \Rightarrow \quad \frac{1}{x} - \frac{5}{4y} = -\frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{x} = -\frac{1}{2} + \frac{5}{4y} \quad \Rightarrow \quad 4y = x(5 - 2y)$$

$$\Rightarrow \quad y = \frac{5x}{4 + 2x} \qquad \ldots (3)$$

Substituting in equation (1), we get

$$\frac{1}{x} - \frac{1}{x-1} + \frac{1}{5x/(4+2x)} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{x-1} + \frac{4+2x}{5x} = \frac{1}{2} \quad \Rightarrow \quad x^2 - 9x + 18 = 0$$

$\Rightarrow \quad x^2 - 6x - 3x + 18 = 0 \quad \Rightarrow \quad x(x - 6) - 3(x - 6) = 0$

$\Rightarrow \quad (x - 3)(x - 6) = 0 \quad \Rightarrow \quad x = 3$ and 6

Given that A take less than 5 hours to fill the tank.

Thus, $x = 3$

Substitute x in equation (3), we get

$$y = \frac{5 \times 3}{4 + (2 \times 3)} = \frac{15}{4+6} = \frac{3}{2} \text{ hours} = 1.5 \text{ hours}$$

$$= 90 \text{ minutes}$$

Hence, time taken by C is 90 minutes.

6. (b) : Let the number of registered votes be $100x$.

The number of votes casted = $80x$

One of the candidate (say candidate 1) got 30 percent of the castes votes

$$= \frac{30}{100} \times 80x = 24x$$

Remaining number of votes = $80x - 24x = 56x$

Other three people got votes in the ratio 1 : 2 : 3. Let name of them be candidate 2, candidate 3 and candidate 4.

$$\text{Candidate 2 got votes} = \frac{1}{1+2+3} \times 56x = \frac{28x}{3}$$

$$\text{Candidate 3 got votes} = \frac{2}{1+2+3} \times 56x = \frac{56x}{3}$$

$$\text{Candidate 4 got votes} = \frac{3}{1+2+3} \times 56x = 28x$$

It is given that the winner of the election received 2512 votes more than the candidate with the second highest votes.

$\Rightarrow \quad 28x - 24x = 2512 \quad \Rightarrow \quad x = 628$

$\therefore$ The number of registered votes = $100x = 62800$

7. (a) : Let the cost price of article B per units be ₹ a.

Cost price of article A per units = ₹ $0.8a$

Selling price of 10 units of article A at 10% profit

$$= 10 \times \frac{110}{100} \times 0.8a = 8.8a$$

Selling price of 20 units of article B at 20% profit

$$= 20 \times \frac{120}{100} \times a = 24a$$

Remaining quantity of article B (120 – 20 = 100) and article A (110 – 10 = 100).

Selling price of the remaining mixture

$= 308.32 \times (100 + 100) = 308.32 \times 200 =$ ₹ 61664

Total cost price = cost price of article A + cost price of article B

$= (110 \times 0.8a) + (120 \times a) = 208a$

Overall profit % = 64%

1.64 × total cost price = total selling price

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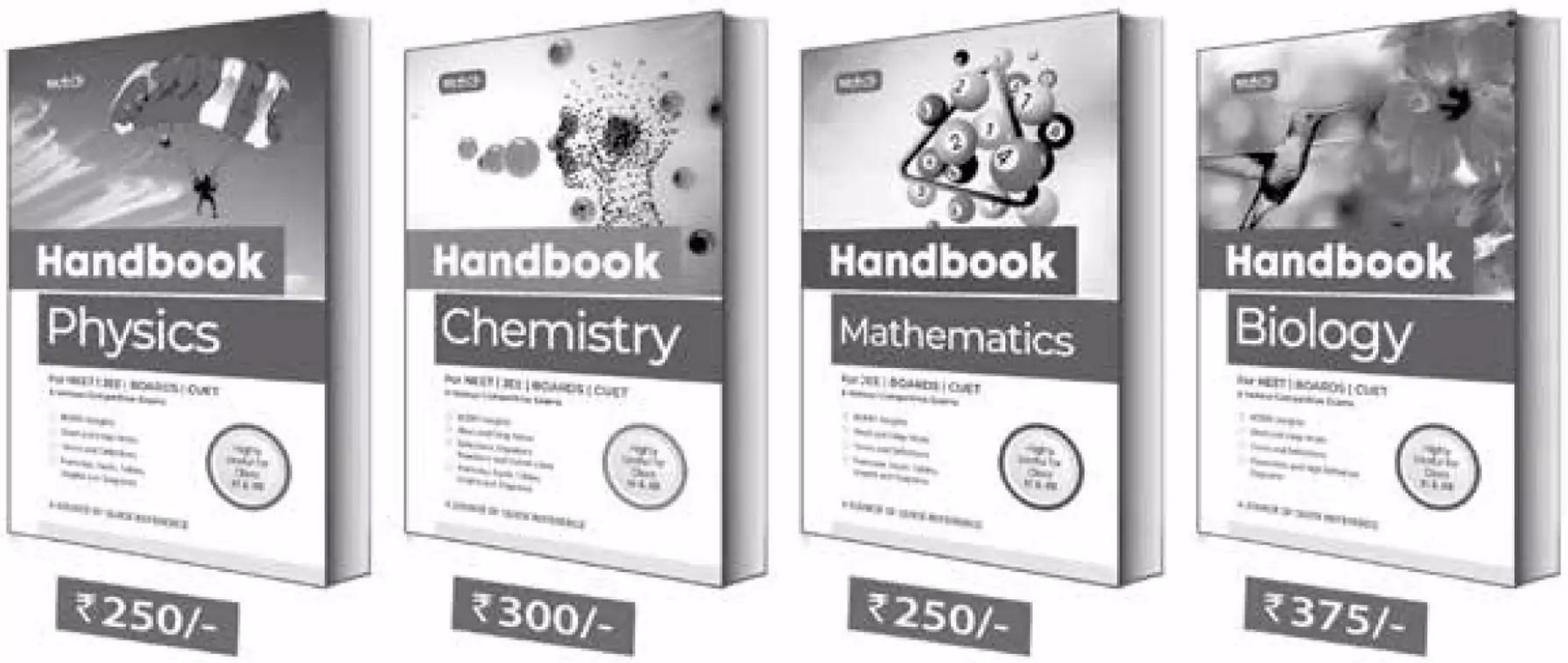

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$\Rightarrow$ $1.64 \times 208a = 8.8a + 24a + 61664$

$\Rightarrow$ $a = 200$

Here, cost price of article A per unit = $0.8a$

$= 0.8 \times 200 = ₹\ 160$

8. (d) : Combined annual interest income

$= 5\%$ of $₹\ 20{,}000 = ₹\ 1000$

Amount invested in bank C

$= 20{,}000 - 8000 - 5000 = ₹\ 7{,}000$

The total interest income is the sum of the interest form all three investments, which is equal to ₹ 1000.

$\therefore$ Interest from A + interest from B + interest from C

$= ₹\ 1000$

$$\Rightarrow \frac{8000 \times 5.5 \times 1}{100} + \frac{5000 \times 5.6 \times 1}{100} + \frac{7000 \times x \times 1}{100} = 1000$$

$\Rightarrow$ $440 + 280 + 70x = 1000$ $\Rightarrow$ $70x = 1000 - 720$

$\Rightarrow$ $70x = 280$ $\Rightarrow$ $x = 4\%$

So, the interest rate for bank C is 4%

The annual interest, she had invested the entire capital in bank $C = 20000 \times \frac{4}{100} = ₹\ 800$

9. (c) : Total score of 40 students = $40 \times 50 = 2000$

Total score except highest & lowest = $38 \times 49 = 1862$

$\Rightarrow$ Highest + Lowest = $2000 - 1862 = 138$

Now, H + L = 138; H – L = 60

By solving, we get, highest score = 99.

10. (d) :

Alloy 1	Copper	Aluminium
$2x$	$3x$	
Alloy 2	Copper	Zinc
	$2y$	$7y$

If aluminium and zinc have to be equal.

Alloy 1	Copper	Aluminium
	14	21
Alloy 2	Copper	Zinc
	6	21

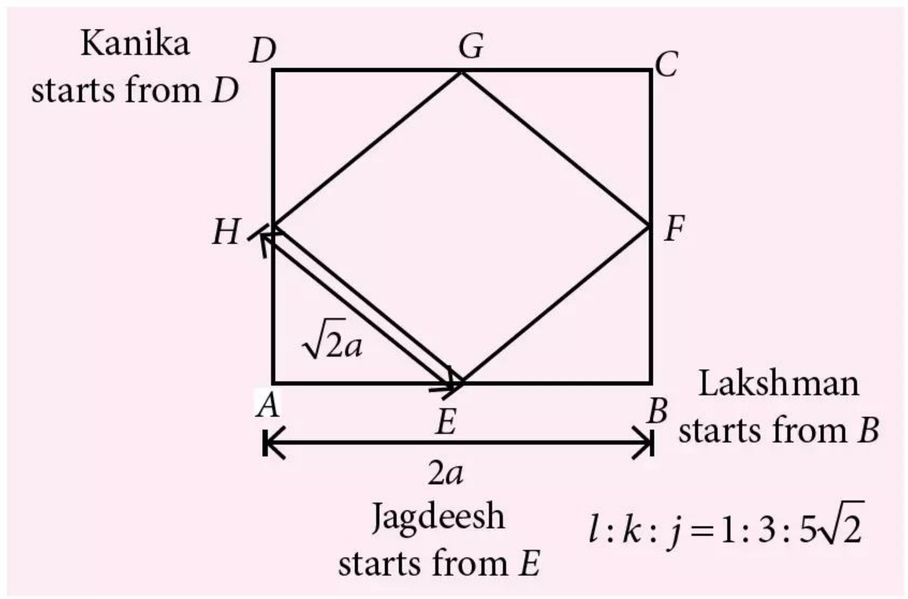
Even a gram of alloy 1 above this level would mean that there is more aluminium than zinc in the total alloy.

The percentage of copper in the total alloy

$$= (14+6) \times \frac{100}{62} = 32.25\%$$

Thus, the percentage of copper ranges from 32.25% to 40% in the alloy.

11. (a) : Lakshman and Kanika start from points B and D respectively at speed l *kmph* and k *kmph* respectively along the sides of the square $ABCD$. They are at distance of $4a$ from each other. Since, they are at diametrically opposite points, the relative distance would be $4a$ irrespective of the directions they choose to travel in. So, to meet for the first time they would have travelled a distance of $4a$ together.



To meet the second time, they would have travelled a further $8a$ together. Essentially, between them, they would have to cover the entire perimeter of the square to meet again.

So, by the time they meet for the second time, they would have covered a distance of $12a$ together. Their speeds are in the ratio 1 : 3. So, Lakshman would have travelled $3a$ and Kanika would have travelled $9a$.

In the same time, Jagdeesh travels along the square $EFGH$ in the anticlockwise direction at j *kmph* and meets Lakshman and Kanika at H, while Jagdeesh meets the other two for the first time.

The ratio of Lakshman's speed to that of Jagdeesh is $1 : 5\sqrt{2}$. So, they would have travelled distances in the same ratio as well.

So, if Lakshman has travelled a distance of $3a$, Jagdeesh should have travelled a distance of $3a \times 5\sqrt{2}$ to reach H.

Hence, the answer is $7.5 \times \sqrt{2}$ times the side of the square $ABCD$.

12. (a) : Let the cost price of article P be ₹ 100.

Marked price (M.P.) = $\frac{120}{100} \times 100 = ₹\ 120$

The shopkeeper then gave a discount of d%, so the selling price (S.P.) after the discount is

$$\text{S.P.} = \text{M.P.} \times \left(1 - \frac{d}{100}\right) \Rightarrow \text{S.P.} = 120 \times \left(1 - \frac{d}{100}\right)$$

The shopkeeper incurred a loss of (d – 11%), so the selling price is

$$\text{S.P.} = \text{C.P.} \times \left(1 - \frac{(d-11)}{100}\right)$$

$$\Rightarrow \quad \text{S.P.} = 100 \times \left(\frac{111-d}{100}\right)$$

Equating the two expressions for the selling price of article P, we have

$$120 \times \left(1 - \frac{d}{100}\right) = 100 \times \left(\frac{111-d}{100}\right) \Rightarrow \quad d = 45$$

Profit on article $Q = 1.5d\% = 1.5 \times 45\% = 67.5\%$

The selling price of article Q

$$= 200 \times \left(1 + \frac{67.5}{100}\right) = ₹\,335$$

13. (b) : Given, Atul's age = 34 years

$\Rightarrow \quad 3p + 2q = 34$ ... (i)

Wife's age = 34 – 5 = 29 years

Son's age = $q + 3$

$\therefore \quad p = q + 3 \Rightarrow p - q = 3$ ... (ii)

Solving (i) and (ii), we get $q = 5$, $p = 8$

$\therefore$ Average age of family $= \frac{34+29+8+5}{4} = \frac{76}{4} = 19$ years

14. (b) : Number of words starting with A $= \frac{5!}{2!} = 60$

Word starting with A where 2D's are together = 4! = 24

Word starting with A, without 2D's adjacent to each other = 60 – 24 = 36

Number of word starting with DA = 4! = 24

Number of word starting with DEA = 3! = 6

Number of word starting with DED = 3! = 6

Then starting with DEM

First word is DEMADN

Second word is DEMAND

$\therefore$ The rank of the word DEMAND

= 36 + 24 + 6 + 6 + 1 + 1 = 74

15. (a) : Remainder when 12 divides 1821, 1823, 1827 are 9, 11 and 3 respectively. Hence; the required remainder will be

$$\left(\frac{9 \times 11 \times 3}{12}\right) = \frac{297}{12}$$

Now, when 297 divided by 12, the remainder is 9.

16. (d) : Number of boys in school F

$$= \frac{7}{11} \times \frac{11}{100} \times 4800 = 7 \times 48 = 336$$

Number of girls in school E

$$= \frac{5}{8} \times \frac{8}{100} \times 4800 = 5 \times 48 = 240$$

So, the ratio of boys in school F to girls in school E = 336 : 240 = 7 : 5

17. (c) : Girls in school D : Boys in school C

$$= \frac{5}{11} \times \frac{22}{100} \times 4800 : \frac{5}{12} \times \frac{16}{100} \times 4800 = 10 : \frac{20}{3} = 3 : 2$$

18. (a) : Total number of students studying in school B

$$= \frac{12}{100} \times 4800 = 12 \times 48$$

Total number of students studying in school E

$$= \frac{8}{100} \times 4800 = 8 \times 48$$

Required percentage $= \frac{12 \times 48}{8 \times 48} \times 100 = 150\%$

19. (c) : Number of girls studying in school D : Number of girls studying in school E

$$= \frac{5}{11} \times 22\% \text{ of } 4800 : \frac{5}{8} \times 8\% \text{ of } 4800 = 10 : 5 := 2 : 1$$

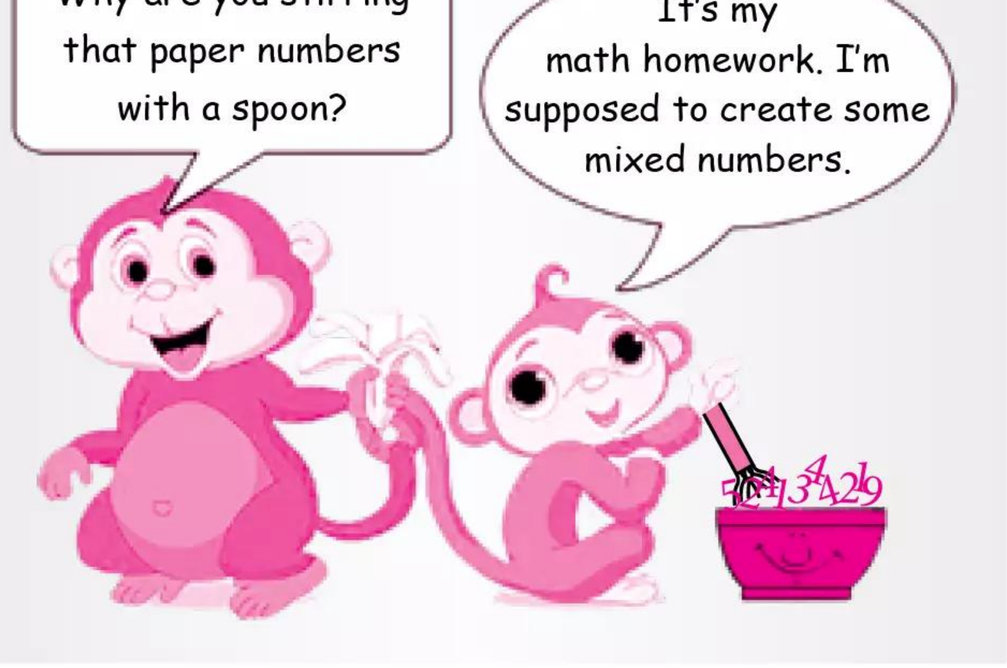
20. (d) : Number of students in School A = 18% of 4800

Number of students in School E = 8% of 4800

Difference = (18 – 8)% of 4800

$\therefore$ Required value $= \frac{10\% \text{ of } 4800}{100\% \text{ of } 4800} \times 4800 = 480$

COMIC CAPSULE



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SECTION-I

Single Option Correct Type

1. The range of $f(x)=4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ is

(a) $[0, 2\pi)$ (b) $[0, \pi]$
(c) $[0, 2\pi]$ (d) $[0, \pi)$

2. If $\cos\theta + i\sin\theta$, $\theta \in R$, is a root of the equation $a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n = 0$, $(a_0, a_1, ... a_n \in \mathbb{R}, a_0 \neq 0)$, then the value of $a_1\sin\theta + a_2\sin 2\theta + ... + a_n\sin n\theta$ is

(a) $2n$ (b) n (c) 0 (d) $n+1$

3. A particle moves along the curve $\frac{x^2}{16}+\frac{y^2}{4}=1$.

When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is

(a) III or IV (b) I or III
(c) II or III (d) II or IV

4. If the total number of m-element subsets of the set $A = \{a_1, a_2, ..., a_n\}$ is k times the number of m element subsets containing a_4, then n is

(a) $(m-1)k$ (b) mk
(c) $(m+1)k$ (d) $(m+2)k$

5. $\lim\limits_{n\to\infty}\left\{\frac{\sqrt{n}}{\sqrt{(n^3)}}+\frac{\sqrt{n}}{\sqrt{(n+4)^3}}+\frac{\sqrt{n}}{\sqrt{(n+8)^3}}+.....+\frac{\sqrt{n}}{\sqrt{[n+4(n-1)]^3}}\right\}$ is

(a) $\frac{5-\sqrt{5}}{10}$ (b) $\frac{5+\sqrt{5}}{10}$
(c) $\frac{2+\sqrt{3}}{2}$ (d) $\frac{2-\sqrt{3}}{2}$

6. Given that a function $f(x) = [x]^2 + x^2$, where $[x]$ is the greatest integer less than or equal to x. If $f(x) > 25$, then the value of x is

(a) any real number
(b) a member of the set $\{x | x \geq 0\}$
(c) a member of the set $\{x | x \leq -4 \text{ or } x \geq 4\}$
(d) a member of the set $\{x | x \geq 25 \text{ or } x \leq 0\}$

7. If the system of linear equations $x - 4y + 7z = g$, $3y - 5z = h$, $-2x + 5y - 9z = k$ is consistent, then

(a) $g + h + k = 0$ (b) $2g + h + k = 0$
(c) $g + h + 2k = 0$ (d) $g + 2h + k = 0$

8. In a certain test, there are n questions. In this test 2^{n-i} students gave wrong answers to at least i questions, where $i = 1, 2, ..., n$. If the total number of wrong answers given is 2047, then n is equal to

(a) 10 (b) 11 (c) 12 (d) 13

9. If $f(x)=\frac{x}{2}-1$, then on the interval $[0, \pi]$

(a) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both continuous

(b) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both discontinuous

(c) $\tan[f(x)]$ is continuous but $\frac{1}{f(x)}$ is not continuous

(d) $\tan[f(x)]$ is not continuous but $\frac{1}{f(x)}$ is continuous

10. The value of $\sum\limits_{k=1}^{13}\frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to

(a) $3-\sqrt{3}$ (b) $2(3-\sqrt{3})$
(c) $2(\sqrt{3}-1)$ (d) $2(2+\sqrt{3})$

SECTION-II

Numerical Answer Type

11. Let $A=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to _____ .

12. Let $f: [-3, 1] \to R$ be given as

$$f(x)=\begin{cases} \min\{(x+6), x^2\}, & -3 \le x \le 0 \\ \max\{\sqrt{x}, x^2\}, & 0 \le x \le 1 \end{cases}.$$

If the area bounded by $y = f(x)$ and x-axis is A, then the value of 6 A is equal to _____.

13. Let $a=3\sqrt{2}$ and $b=\dfrac{1}{5^{1/6}\sqrt{6}}$. If $x, y \in \mathbb{R}$ are such that $3x+2y=\log_a(18)^{5/4}$ and $2x-y=\log_b(\sqrt{1080})$, then $4x + 5y$ is equal to _____.

14. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors. The angle between $\vec{a}$ and $\vec{b}$ is 30°, the angle between $\vec{a}$ and $\vec{c}$ is 60° and the angle between $\vec{a}$ and $\vec{b}+\vec{c}$ is 45°. If $|\vec{b}|=\sqrt{6}$ and $|\vec{c}|=2\sqrt{2}$, then $|\vec{b}+\vec{c}|=$ _____.

15. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is _____.

SOLUTIONS

1. (a): Given, $4\sin^{-1}\left(\dfrac{x^2}{x^2+1}\right)$

Now, $\dfrac{x^2}{x^2+1}=\dfrac{x^2+1-1}{x^2+1}=1-\dfrac{1}{x^2+1}$

$x^2 \ge 0; x^2 + 1 \ge 1 \therefore \quad 0<\dfrac{1}{x^2+1}\le 1 \Rightarrow -1\le\dfrac{-1}{x^2+1}<0;$

$0\le 1-\dfrac{1}{x^2+1}<1;\ 0\le 4\sin^{-1}\left(\dfrac{x^2}{x^2+1}\right)<2\pi$

2. (c): $a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n = 0$

$\Rightarrow \quad a_0 + a_1x^{-1} + ... + a_{n-1}x^{1-n} + a_nx^{-n} = 0$

Now, $\cos\theta + i\sin\theta = e^{i\theta}$ is the root of the equation, we get

$\Rightarrow \quad a_0 + a_1e^{-i\theta} + a_2e^{-2i\theta} + ... + a_{n-1}e^{-i(n-1)} + a_ne^{-in\theta} = 0$

$\Rightarrow \quad (a_0 + a_1\cos\theta + a_2\cos 2\theta + ... + a_n\cos n\theta)$
$-i(a_1\sin\theta + a_2\sin 2\theta + ... + a_n\sin n\theta) = 0$
$\therefore \quad a_0 + a_1\cos\theta + a_2\cos 2\theta + ... + a_n\cos n\theta = 0$
and $\quad a_1\sin\theta + a_2\sin 2\theta + ... + a_n\sin n\theta = 0$

3. (d): We have, $\dfrac{x^2}{16}+\dfrac{y^2}{4}=1$ ...(i)

and $\dfrac{dx}{dt}=\dfrac{4dy}{dt}$

Differentiating (i) w.r.t. t,

we get $\dfrac{2x}{16}\dfrac{dx}{dt}+\dfrac{2y}{4}\dfrac{dy}{dt}=0$

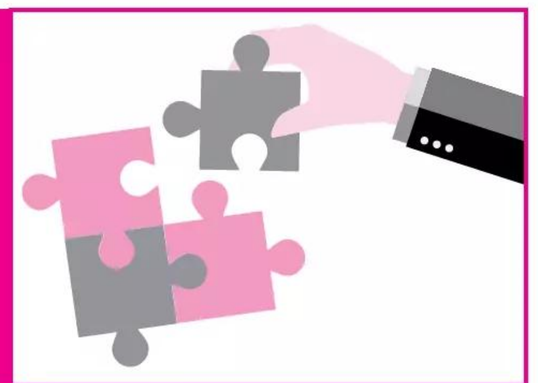
$\Rightarrow \quad \dfrac{2}{16}x\times\dfrac{4dy}{dt}+\dfrac{1}{2}y\dfrac{dy}{dt}=0$

$\Rightarrow \quad \dfrac{x}{2}\dfrac{dy}{dt}=-\dfrac{1}{2}y\dfrac{dy}{dt}$

$\Rightarrow \quad x = -y$ or $y = -x$

$\therefore$ The particle lies in quadrant II or IV.

PUZZLE CORNER



MATHDOKU

Fill the grid with the numbers 1 to 8 such that each number appears only once in each row and column. The Grid is also divided in outlined regions called cages each with a given operator and target number. The numbers in the individual cells of a cage must produce that target number using the operator in a mathematical calculation.

4–	5–		4÷		1–		16+
	14×		2÷	28×		5	
20+	640×				4×		
			14×	2–	9×		12+
	15+						
	5	4×		90×		336×	
15+	1–						23+
			1–				

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' names with their valuable feedback will be published in next issue.

4. (b): Total number of elements in set $A = n$

According to given condition, ${}^{n}C_{m} = k\,{}^{n-1}C_{m-1}$

$$\Rightarrow \quad \frac{{}^{n}C_{m}}{{}^{n-1}C_{m-1}} = k \quad \Rightarrow \quad k = \frac{n!\ (m-1)!\ (n-m)!}{m!\ (n-m)!\ \ (n-1)!}$$

$$\Rightarrow \quad n = mk$$

5. (a): Let

$$L = \lim_{n\to\infty}\left[\frac{\sqrt{n}}{\sqrt{n^3}} + \frac{\sqrt{n}}{\sqrt{(n+4)^3}} + \ldots + \frac{\sqrt{n}}{\sqrt{(n+4(n-1))^3}}\right]$$

$$= \lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{\sqrt{n}}{\sqrt{(n+4r)^3}} = \lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{n\sqrt{\left(1+\frac{4r}{n}\right)^3}}$$

$$= \int_0^1 \frac{dx}{\sqrt{(1+4x)^3}} = \left[\frac{(1+4x)^{\frac{-3}{2}+1}}{\left(\frac{-3}{2}+1\right)4}\right]_0^1$$

$$= \frac{-1}{2}\left(5^{-1/2}-1\right) = \frac{1}{2}\left(1-\frac{1}{\sqrt{5}}\right) = \frac{5-\sqrt{5}}{10}$$

6. (c): We have, $f(x) = [x]^2 + x^2$

Given, $f(x) > 25$

$\therefore \quad [x]^2 + x^2 > 25$

$\Rightarrow (x - \{x\})^2 + x^2 > 25$

$[\because x = [x] + \{x\}$ where $0 \le \{x\} < 1]$

$\Rightarrow x^2 + \{x\}^2 + x^2 - 2x\{x\} > 25$

$\Rightarrow 2x^2 - 2x\{x\} + \{x\}^2 > 25$

$\Rightarrow 2x(x - \{x\}) + \{x\}^2 > 25$

$\Rightarrow 2x[x] + \{x\}^2 > 25$

$\Rightarrow 2x[x] > 24 \qquad [\because 0 \le \{x\}^2 < 1]$

$\Rightarrow x[x] > 12$, which is possible for $x > 4$ or $x \le -4$

$\therefore$ Value of x is the member of the set

$\{x \mid x \le -4$ or $x \ge 4\}$

7. (b): We have, $\Delta = \begin{vmatrix} 1 & -4 & 7 \\ 0 & 3 & -5 \\ -2 & 5 & -9 \end{vmatrix}$

$= 1(-27 + 25) + 0 - 2(20 - 21) = -2 + 2 = 0$

$\therefore$ System of equations will be consistent if

$\Delta_x = \Delta_y = \Delta_z = 0$

$$\therefore \quad \Delta_z = \begin{vmatrix} 1 & -4 & g \\ 0 & 3 & h \\ -2 & 5 & k \end{vmatrix} = 0$$

$\Rightarrow 1(3k - 5h) - 2(-4h - 3g) = 0$

$\Rightarrow 3k - 5h + 8h + 6g = 0$

$\Rightarrow 2g + h + k = 0$

Similarly, $\Delta_x = \Delta_y = 0$

$\Rightarrow 2g + h + k = 0$

8. (b): Total number of questions = n

Number of students who gave wrong answers to atleast i questions = 2^{n-i}

Number of students who gave wrong answers to atleast $(i + 1)$ questions = $2^{n-(i+1)}$

$\therefore$ Number of students who gave wrong answers to exactly 'i' questions = $2^{n-i} - 2^{n-(i+1)}$

Now number of students who gave 1 wrong answers $= 1(2^{n-1} - 2^{n-2})$

Number of students who gave 2 wrong answers $= 2(2^{n-2} - 2^{n-3})$

Now number of students who gave 3 wrong answer $= 3(2^{n-3} - 2^{n-4})$

Number of students who gave all wrong answer = $n(2^0)$

$\therefore$ Total number of wrong answers

$= 1(2^{n-1} - 2^{n-2}) + 2(2^{n-2} - 2^{n-3}) + 3(2^{n-3} - 2^{n-4}) + \ldots + n(2^0)$

$= 2^{n-1} + 2^{n-2} + 2^{n-3} + \ldots\ldots + 2^0 = 2047$ [Given]

$$\Rightarrow \quad 1\left(\frac{2^n - 1}{2-1}\right) = 2047$$

$\Rightarrow 2^n - 1 = 2047 \Rightarrow 2^n = 2048 = 2^{11}$

$\therefore \quad n = 11$

9. (c): $f(x) = \frac{x}{2} - 1 = \frac{x-2}{2} \Rightarrow \frac{1}{f(x)} = \frac{2}{x-2}$

and $\tan[f(x)] = \tan\left[\frac{x-2}{2}\right]$

Here, $\frac{1}{f(x)}$ is not defined, when, $x - 2 = 0$

i.e., $\frac{1}{f(x)}$ is not continuous at $x = 2 \in [0, \pi]$

Also, $\tan\theta$ is discontinuous at $\theta = \frac{-\pi}{2}$ and $\frac{\pi}{2}$

$\therefore \quad \frac{x-2}{2} = \frac{\pi}{2} \Rightarrow x = 2 + \pi \notin [0, \pi]$

and $\frac{x-2}{2} = \frac{-\pi}{2} \Rightarrow x = 2 - \pi \notin [0, \pi]$

So, $\tan[f(x)]$ is continuous when $x \in [0, \pi]$.

10. (c) : $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$

$$=\sum_{k=1}^{13} \frac{\sin\left\{\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)-\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\right\}}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)} \cdot \frac{1}{\sin\frac{\pi}{6}}$$

$$=2\sum_{k=1}^{13}\left[\cot\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\right]$$

$$=2\left[\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right]=2\left[\cot\frac{\pi}{4}-\cot\frac{5\pi}{12}\right]$$

$$=2[1-(2-\sqrt{3})]=2(-1+\sqrt{3})=2(\sqrt{3}-1)$$

11. (10) : We have, $A=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$$\therefore \quad A^2=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}=\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

So, $A^4=\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$$=\begin{bmatrix} x^4+1+3x^2 & x^3+2x \\ x^3+2x & x^2+1 \end{bmatrix}$$

Now, $a_{11} = 109$

$\Rightarrow x^4 + 3x^2 + 1 = 109$

$\Rightarrow x^4 + 3x^2 - 108 = 0$

$\Rightarrow x^4 + 12x^2 - 9x^2 - 108 = 0$

$\Rightarrow (x^2 + 12)(x^2 - 9) = 0$

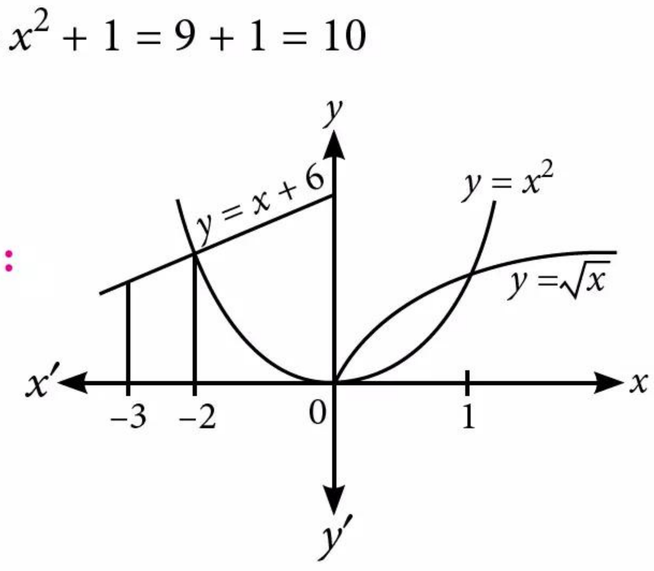
$\Rightarrow x^2 = -12$ (not possible)

$\therefore \quad x^2 = 9$

$\Rightarrow x = \pm 3$

So, $a_{22} = x^2 + 1 = 9 + 1 = 10$

12. (41) :



Area bounded by $y = f(x)$ and x-axis,

$$A=\int_{-3}^{-2}(x+6)\,dx+\int_{-2}^{0}x^2\,dx+\int_{0}^{1}\sqrt{x}\,dx$$

$$=\left[\frac{x^2}{2}+6x\right]_{-3}^{-2}+\left[\frac{x^3}{3}\right]_{-2}^{0}+\left[\frac{x^{3/2}}{3/2}\right]_{0}^{1}=\frac{7}{2}+\frac{8}{3}+\frac{2}{3}$$

$$\therefore \quad A=\frac{41}{6} \Rightarrow 6A = 41$$

13. (8) : $3x+2y=\log_{3\sqrt{2}}(3\sqrt{2})^{5/2}=\frac{5}{2}$

Also, $2x-y=\log_{\frac{1}{5^{1/6}\sqrt{6}}}(5^{1/6}\cdot\sqrt{6})^3=-3$

$4x + 5y = 2(3x + 2y) - (2x - y)$

$$=2\cdot\frac{5}{2}-(-3)=5+3=8$$

14. (5) : $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

$\Rightarrow |\vec{a}||\vec{b}+\vec{c}|\cos 45^\circ=|\vec{a}||\vec{b}|\cos 30^\circ+|\vec{a}||\vec{c}|\cos 60^\circ$

$$\Rightarrow |\vec{b}+\vec{c}|\frac{1}{\sqrt{2}}=\sqrt{6}\times\frac{\sqrt{3}}{2}+2\sqrt{2}\times\frac{1}{2}$$

$$\Rightarrow |\vec{b}+\vec{c}|=\frac{6}{2}+\frac{4}{2}=3+2=5$$

15. (3) : Given curves are $\frac{x^2}{9}+\frac{y^2}{4}=1$ and $x^2+y^2=\frac{31}{4}$

Let slope of common tangent be m.

So, tangents of given ellipse and circle are respectively

$$y=mx\pm\sqrt{9m^2+4}$$

and $y=mx\pm\frac{\sqrt{31}}{2}\sqrt{1+m^2}$

Hence, $9m^2+4=\frac{31}{4}(1+m^2)$

$\Rightarrow 36m^2 + 16 = 31 + 31m^2$

$\Rightarrow 5m^2 = 15$

$\Rightarrow m^2 = 3$

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(b)	**2.**	(c)	**3.**	(a)	**4.**	(c)	**5.**	(b)
6.	(d)	**7.**	(a,b,c)	**8.**	(c)	**9.**	(a,b,c)	**10.**	(a,c)
11.	(a,c,d)	**12.**	(a,c,d)	**13.**	(a,d)	**14.**	(d)	**15.**	(a)
16.	(a)	**17.**	(3)	**18.**	(8)	**19.**	(1)	**20.**	(2)

GK CORNER

Enhance Your General Knowledge with Current Updates!

STATE NEWS

1 In a recent development, Kandula Durgesh, the Minister for Tourism and Cinematography, announced a significant funding allocation for two major tourism projects in Andhra Pradesh. The focus of this funding is to enhance the Akhanda Godavari Project and Gandikota tourism circuits. These projects will be carried out under the Public Private People Partnership (4P) model, with tenders expected to be invited soon.

2 In a significant move, Assam Chief Minister, Himanta Biswa Sarma recently declared that Dibrugarh will be upgraded to a full-fledged city and will serve as the second capital of Assam. This decision is aimed at improving the Ease of Living and boosting connectivity in the region. The announcement, made during the Republic Day celebrations, marks a transformative shift for city once associated with militancy.

3 The Odisha government is launching a new project to empower local communities in forest management. Under a fresh phase of the Forest Rights Act, 100 villages in the state will receive direct funding into their bank accounts. This initiative aims to strengthen forest protection and management by promoting local governance through gram sabhas (village councils).

4 The Uttar Pradesh government has introduced a comprehensive Aerospace and Defence Unit policy designed to attract investments and create employment opportunities. This initiative aims to generate significant investments and jobs over the next five years, positioning Uttar Pradesh as a key hub in the aerospace and defence sector.

5 On January 20, 2025, Chhattisgarh Chief Minister, Vishnu Deo Sai launched 'Deendayal Upadhyay Bhoomiheen Krishi Majdoor Kalyana Yojana' in Raipur. This initiative is designed to provide financial aid of ₹10,000 annually to over 5.62 lakh landless agricultural labourers, Baigas and Gunias. The scheme has already disbursed more than ₹562 crores to its beneficiaries.

6 The Kerala government has recently rolled out an extensive education plan aimed at improving the quality of public schools. The initiative follows a meeting chaired by Chief Minister Pinarayi Vijayan, emphasizing accountability and the timely execution of educational reforms. The plan outlines eight key areas for immediate focus, targeting improvements for both the current and upcoming academic years.

7 Uttarakhand has become the first state in India to approve a comprehensive manual for the Uniform Civil Code (UCC), aiming to standardize personal laws across the state. The initiative will streamline processes related to marriage, divorce, inheritance, and more, with training for officials and online services set to launch soon.

8 The Government of India has approved a major financial package to revive the Visakhapatnam Steel Plant (VSP) in Andhra Pradesh, aimed at strengthening the steel industry and boosting

regional economic growth. Chief Minister N. Chandrababu Naidu expressed gratitude to Prime Minister Narendra Modi deeming this decision a landmark event for the state.

9 The Maharashtra government has set up a 16-member committee to draft the state's first Artificial Intelligence (AI) policy, aiming to position the state as a leader in AI development. The policy will align with the central government's India AI Mission, boosting the state's digital economy and industrial growth.

10 Union Minister for Education, Dharmendra Pradhan, recently launched the registration portal for Kashi Tamil Sangamam (KTS) 3.0. The event, scheduled from 15th to 24th February 2025 in Varanasi, Uttar Pradesh, aims to strengthen cultural ties between Tamil Nadu and Kashi, celebrating their shared heritage.

Test Yourself!

1. What is the main objective of the Kashi Tamil Sangamam (KTS) initiative?
(a) To promote tourism between Tamil Nadu and Uttar Pradesh
(b) To strengthen cultural ties between Tamil Nadu and Kashi
(c) To organize educational seminars in Varanasi
(d) To launch new infrastructure projects in Varanasi

2. Which government launched a new project to empower local communities in forest management?
(a) Uttar Pradesh (b) Himachal Pradesh
(c) Odisha (d) Andhra Pradesh

3. Who is the Minister for Tourism and Cinematography who announced funding for two major tourism projects in Andhra Pradesh?
(a) N. Chandrababu Naidu
(b) Kandula Durgesh
(c) Pinarayi Vijayan
(d) Himanta Biswa Sarma

4. Which city in Assam will be upgraded to a full-fledged city and serve as the second capital of Assam?
(a) Guwahati (b) Dibrugarh
(c) Jorhat (d) Silchar

5. Which state has introduced a new policy for Aerospace and Defence Unit to attract investments?
(a) Uttar Pradesh (b) Gujarat
(c) Maharashtra (d) Haryana

6. What is the main target of the education plan rolled out by the Kerala government?
(a) Reducing student dropout rates
(b) Enhancing public school quality
(c) Increasing private school enrollment
(d) Promoting online education

7. Which state became the first in India to approve a comprehensive manual for the Uniform Civil Code (UCC)?
(a) Maharashtra (b) Karnataka
(c) Uttarakhand (d) Uttar Pradesh

8. What is the expected outcome of the major financial package approved for the Visakhapatnam Steel Plant (VSP)?
(a) Strengthening the steel industry and regional economic growth
(b) Reducing pollution in the region
(c) Expanding agricultural exports
(d) Developing a new port in Visakhapatnam

9. What does the Maharashtra AI policy aim to align with?
(a) National Digital Literacy Mission
(b) India's Smart Cities Mission
(c) Central government's India AI Mission
(d) Make in India initiative

10. Which communities are specifically mentioned as beneficiaries of the 'Deendayal Upadhyay Bhoomiheen Krishi Majdoor Kalyana Yojana' in Chhattisgarh?
(a) Baigas and Gunias
(b) Kurmis and Gonds
(c) Bhils and Santhals
(d) Rajputs and Brahmins

Answer Key

1. (b)	2. (c)	3. (b)	4. (b)	5. (a)	6. (b)	7. (c)	8. (a)	9. (c)	10. (a)

Infinite Learning

Trigonometry : The Heartbeat of Mathematics

In a world of angles, waves and circular motion, one mathematical field rules them all: Trigonometry. From architects designing buildings to engineers working on space missions, trigonometry is the unsung hero that helps make the world function in precise and predictable manner. Trigonometry is a key that opens doors to understanding a wide range of phenomena in the world.

In this article, we'll dive deeper into trigonometry's historical roots, its real-world applications and why it remains an essential part of modern mathematics and science.

The Origins of Trigonometry: A Brief History

Trigonometry didn't originate in a single place or time. It evolved across several ancient cultures as humans sought to measure and understand the world around them.

- **Ancient Egyptians and Babylonians:** The earliest forms of trigonometry can be traced back to around 2000 BCE. Ancient Egyptians and Babylonians used basic geometric principles to measure distances and angles, often for construction purposes, such as the building of pyramids or creating accurate maps.
- **Greek Mathematicians:** The term "trigonometry" came from the Greek word "*trigonon*" (means triangle) and "*metron*" (means measure). Pioneers like Hipparchus (190–120 BCE) and Ptolemy (100–170 CE) developed trigonometric tables to help astronomers calculate the positions of stars and planets. These early trigonometric techniques were crucial in advancing astronomy.

The Three Primary Functions: Beyond the Basics

At first glance, the three main trigonometric functions—sine, cosine, and tangent—might seem like abstract concepts, but each plays a critical role in understanding the world around us. Here's a deeper dive into these functions:

- **Sine and Cosine:** The sine and cosine functions are not just about calculating lengths in a right triangle; they are periodic functions that describe wave-like motions. This makes them crucial in understanding oscillations, sound waves, and light waves. A wave's amplitude, frequency, and phase shift are all captured using sine and cosine functions.
- **Tangent:** While sine and cosine focus on the ratio of sides in a right triangle, tangent describes the ratio of sine to cosine. Tangent functions are often used in determining slopes, such as calculating the steepness of a hill or an incline in engineering design. They also appear in calculus, where the slope of a curve at any given point is determined using derivatives.
- **Secant, Cosecant, and Cotangent:** These "secondary" functions—secant, cosecant and cotangent—are simply the reciprocals of cosine, sine and tangent, respectively. While they don't have as prominent a role as the primary functions, they appear in specific applications, particularly in higher mathematics like calculus, and in solving certain types of equations.

The Unit Circle: A Gateway to Advanced Trigonometry

One of the most important tools in trigonometry is the unit circle, a circle with a radius of one unit. Located in the coordinate plane, it provides a powerful way to understand the relationships between angles and trigonometric functions. By measuring the angle in radians from the positive x-axis, the coordinates of any point on the unit circle correspond to the sine and cosine of that angle.

This concept leads to deeper insights into how trigonometric functions behave for all angles, not just those in right triangles.

Trigonometry's Role in Real World

Trigonometry has a multitude of applications in both theoretical and practical fields. While trigonometry's roots lie in geometry, its

applications extend far beyond basic triangle calculations. Here, are just a few of the ways trigonometry is used in the real world:

- **Architecture:** Designing structures, calculating angles for roofs, bridges, and buildings, also ensuring the stability of tall skyscrapers relies on trigonometric principles.
- **Astronomy:** Early astronomers used trigonometry to measure the distance between planets and stars, a technique still used in modern-day space exploration.
- **Navigation:** Whether by sea, air, or land, trigonometry is essential for determining position and course. GPS systems, for instance, rely on advanced trigonometric algorithms to pinpoint your location.
- **Engineering and Physics:** From the study of sound waves to electromagnetic fields and mechanical structures, trigonometry helps explain how systems work and solve real-world problems.
- **Computer Graphics and Animation:** In computer graphics, trigonometry is used to transform shapes, rotate objects, and project three-dimensional images onto a two-dimensional screen.

Trigonometry in Understanding the Future of Mathematics

Trigonometry remains a cornerstone of modern mathematics and science. In fields like signal processing, machine learning and quantum computing, trigonometric functions are used to process complex data, model systems, and analyse signals. For example, in Fourier analysis, trigonometric functions are used to decompose complex signals into simpler components.

As technology advances, the role of trigonometry in solving new problems and creating innovations will only grow. Whether it's optimizing algorithms in artificial intelligence or exploring the fundamental nature of the universe, trigonometry will continue to play a pivotal role in unlocking new discoveries.

Conclusion: The Enduring Relevance of Trigonometry

Though often considered a challenge by students, trigonometry is far more than just a mathematical subject. It's a powerful toolkit that allows us to understand and manipulate the world around us. Whether designing skyscrapers, exploring outer space, or developing the next big innovation in technology, trigonometry is at the heart of countless discoveries and advances.

So, the next time you encounter a sine or cosine function, take a moment to appreciate just how far-reaching and essential trigonometry really is. Its influence is present in nearly every aspect of our modern world, and understanding it opens up new possibilities for exploration and innovation.

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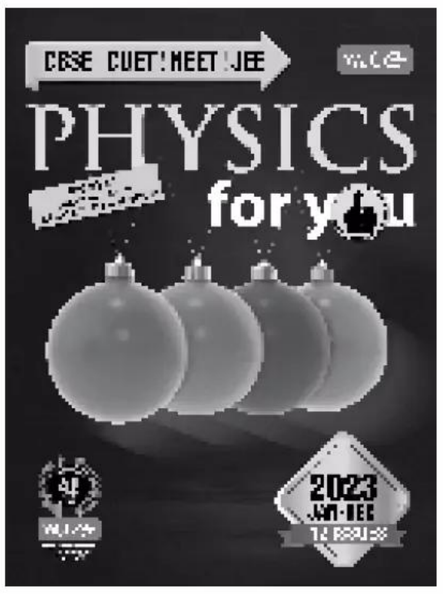

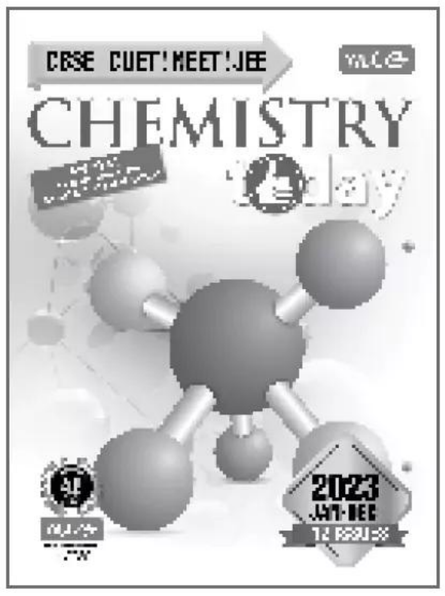

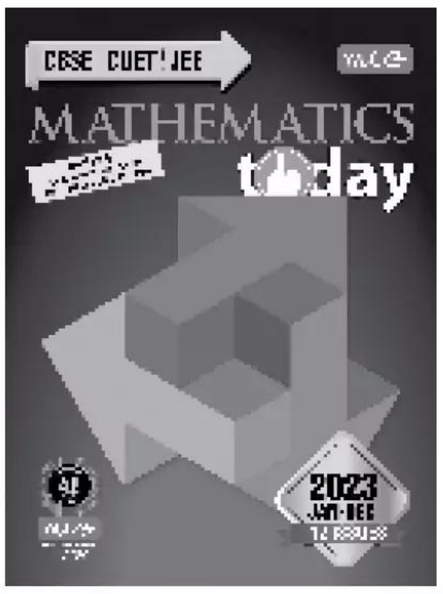

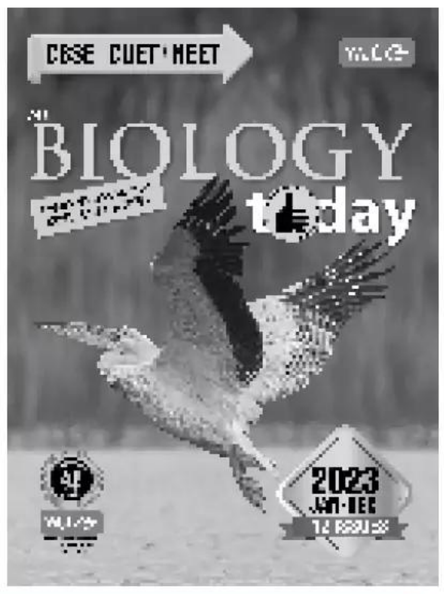

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Brain Teaser

LOGICAL REASONING

For Various Competitive Exams

1. What will come in place of '?' in the given series
2, 7, 24, 77, ?
(a) 238 (b) 1335 (c) 249 (d) 1016

2. Find the wrong term in the given series.
13, 25, 40, 57, 79, 103, 130
(a) 25 (b) 40 (c) 57 (d) 79

Direction Q.(3 and 4) : Read the following information carefully and answer the questions given below :
In a certain coded language, RAT is coded as 1521; LION is coded as 2500; TIGER is coded as 3481.

3. What is the code of MOUSE?
(a) 3249 (b) 4900 (c) 5329 (d) 5184

4. What is the code of JACKAL?
(a) 1296 (b) 1444 (c) 1521 (d) 1600

5. Select the odd one out.
(a) See : Eyes (b) Hear : Ears
(c) Touch : Skin (d) Tongue : Taste

Direction Q.(6 and 7) : Study the following alphabet-symbol-digit sequence and answer the questions given below:
G N = 8 L @ " C R β ↑ T 2 5 H # 7 E J $ 4 + V Y 3 6 ? F 9 * M

6. How many such symbols are there in the above arrangement each of which is immediately preceded by a letter and immediately followed by a number?
(a) Nil (b) Three (c) Four (d) Five

7. How many such letters are there in the above arrangement each of which is either immediately preceded by a number or immediately followed by a symbol?
(a) Three (b) Two (c) Five (d) Four

Direction Q.(8 and 9) : In each questions below are given two statements followed by some conclusions. You have to take the given statements to be true even if they seem to be at variance from commonly known facts and then decide which of the given conclusions logically follows from the given statements, disregarding commonly known facts.
(a) Only conclusion (I) follows.
(b) Only conclusion (II) follows.
(c) Either (I) or (II) follows.
(d) Neither (I) nor (II) follows.

8. **Statements:** Some stars are planets.
No planet is a moon.
Conclusions:
I. No star is a moon.
II. Some stars are moons.

9. **Statements:** Some circles are rectangles.
All squares are rectangles.
Conclusions :
I. At least some squares are circles.
II. All rectangles are circles.

Direction Q.(10 and 11) : Each of the questions given below consists of a question and three statements numbered (I), (II) and (III) given below it. You have to decide whether the data provided in the statements are sufficient to answer the question or not.

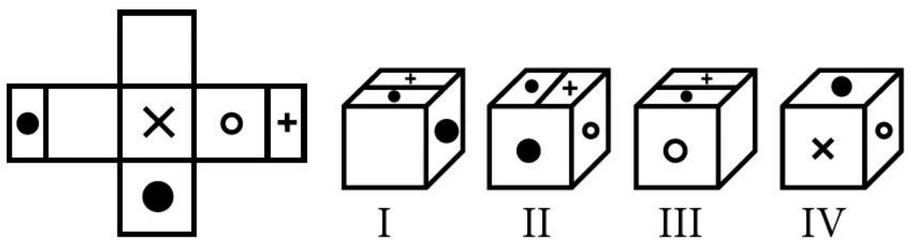
10. What is the rank of Animesh in a class of 17 students?
I. Nirmal who is thirteenth from the bottom is six rank ahead of Bhumika who is two position below Animesh.
II. Bhumika is four position ahead of Kamal.
III. Bhumika is two position below Animesh and Kamal's rank is 15th.
(a) Only (I) and (III) (b) Only (I) and (II)
(c) Only (I) or (II) and (III) together are required to answer the question
(d) Only (II) is required to answer the question

11. Who among A, B, C, D, E and F each having a different height, is the tallest?
I. B is taller than A but shorter than E.
II. Only two of them are shorter than C.
III. D is taller than only F
(a) Only (I) and (II) (b) Only (I) and (III)
(c) Only (II) and (III)
(d) All (I), (II) and (III) are required to answer the question

12. A cube has six faces each of a different colour. The red face is opposite to black. The green face is in between red and black. The blue face is adjacent to white and brown face is adjacent to blue. The four colours adjacent to green are

(a) red, black, brown and white
(b) red, black, brown and blue
(c) red, black, blue and white
(d) red, brown, blue and white

13. In the question given below an unfloded dice is given in the left side while in the right side, four answer choices are given in the form of complete dices. You are required to selected the correct answer choice(s) which is/are formed by folding the unfolded dice.



(a) I and II (b) II, III and IV
(c) I, II, III and IV (d) I, II and III

Direction Q.(14 and 15) : Read the following information carefully and answer the questions that follow:

Point H is 6 m towards the East of point G, point R is 8 m North of point G. Point Q is exactly midway between point R and point G. Point K is 10 m to the South of point Q. Point L is 3 m towards the East of point Q. Point S is exactly midway between point G and point H.

14. If a persons walks 4 m towards the South from point L, takes a right turn and walks for another 3 m, then which of the following points would he reach?
(a) Q (b) G (c) K (d) H

15. If a person walks 8 m towards North from Points S, then which of the following points would he cross and how far will he be from point R?
(a) G, 4 m (b) H, 3 m (c) L, 6 m (d) L, 3 m

Direction Q.(16 to 20) : Study the following information carefully and answer the questions given below:

Eight persons – *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H* are travelling in three different cars namely *X*, *Y* and *Z*. But not necessarily in the same order. There are at least two persons in each car and each car has persons of both the sexes. Out of eight persons three are females. All of them like different colours viz., Red, Green, Yellow, Blue, Black, White, Grey and Purple but not necessarily in the same order. One female does not like Grey or White colour. *D* is travelling with *G* in the car *Z*. *G* likes Black colour. The persons who like Red and Purple colours, are travelling in the same car. *E* does not like Purple colour and he is not travelling with *H* in the same car. *A* does not like purple nor Red colour. *H* is travelling in the car *Y*. *B*, *C* and *H* are females in the group. *B* likes Blue colour and travels with the person who likes Black colour. Red and Green colours are liked by female members. The person who is travelling with *H* likes Grey colour. One of the persons travelling in car Z likes Yellow colour.

16. In which of the following Cars only two persons are travelling?
(a) Car *X* (b) Car *Y*
(c) Car *Z* (d) Cannot be determined

17. The female member who is travelling in the Car *X* likes which colour?
(a) Green (b) Blue (c) Yellow (d) Red

18. Who among the following likes Yellow colour?
(a) *D* (b) *E* (c) *A* (d) *F*

19. Who among the following is/are travelling with female member *C*?
(a) *E* and *G* (b) *A* and *D*
(c) *E* and *F* (d) *D* and *E*

20. Which of the following combinations of Person-Car-Colour is not correct?
(a) *C* - *X* - Red (b) *F* - *X* - Purple
(c) *G* - *Z* - Black (d) *E* - *Y* - Grey

SOLUTIONS

1. (a) : $2 \xrightarrow{2\times3+1} 7 \xrightarrow{7\times3+3}$
$24 \xrightarrow{24\times3+5} 77 \xrightarrow{77\times3+7} \boxed{238}$

2. (c) : $13 \xrightarrow{+(3\times4)} 25 \xrightarrow{+(3\times5)} 40 \xrightarrow{+(3\times6)}$
$\boxed{58} \xrightarrow{+(3\times7)} 79 \xrightarrow{+(3\times8)} 103 \xrightarrow{+(3\times9)} 130$

$\therefore$ Wrong term is 57.

(3 – 4) : Note that, $(RAT)^2 = (18 + 1 + 20)^2 = (39)^2 = 1521$
Similarly, $(LION)^2 = (12 + 9 + 15 + 14)^2 = (50)^2 = 2500$
and $(TIGER)^2 = (20 + 9 + 7 + 5 + 18)^2 = (59)^2 = 3481$

3. (c) : $\therefore (M + O + U + S + E)^2$
$= (13 + 15 + 21 + 19 + 5)^2 = (73)^2 = 5329.$

4. (b) : $\therefore (J + A + C + K + A + L)^2$
$= (10 + 1 + 3 + 11 + 1 + 12)^2 = (38)^2 = 1444.$

5. (d)

6. (b) :

Letter	Symbol	Number

Such combinations are:

N = 8	;	H # 7	;	J $ 4

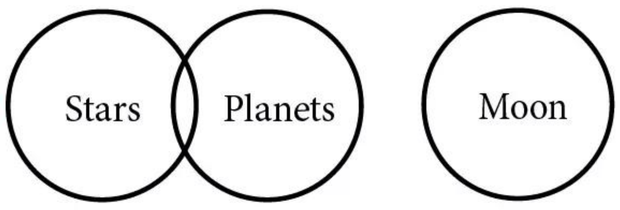
7. (b) :

Number	Letter	Symbol

Such combinations are:

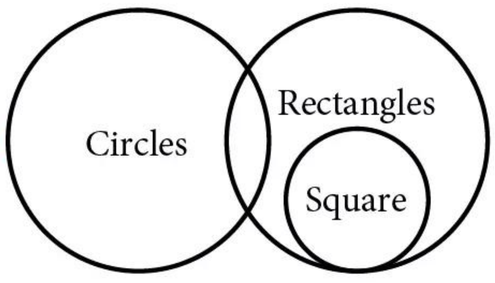
8 L @	;	5 H #

8. (c) :



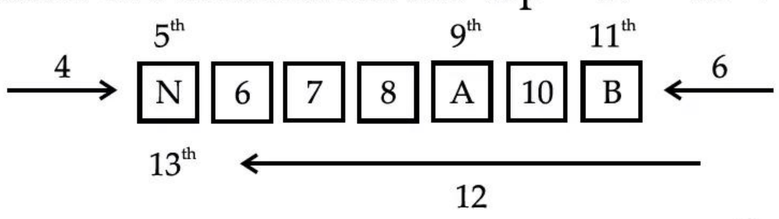
Stars may or may not be moon. Thus, either conclusion (I) or (II) follows.

9. (d) :



Square may or may not be circle thus conclusion (I) is not valid. All rectangle may or may not be circle thus conclusion (II) is not valid. Thus, none of the conclusion follows.

10. (c) : From statement (I),
the position of Nirmal from the top = 17 – 13 + 1 = 5^{th}

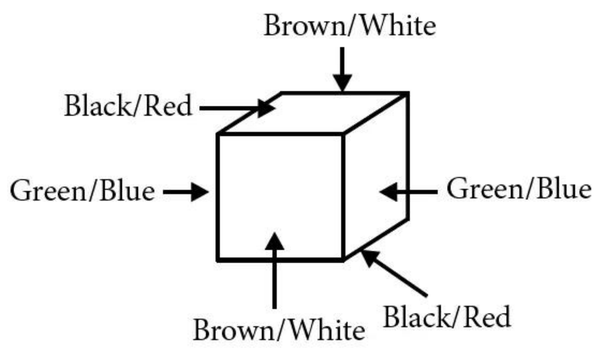


The rank of Animesh from the either end is 9^{th}.
From statement (II), ... B ||| K .....
From statement (III), the rank of Kamal = 15^{th}
Bhumika is two position below Animesh.
From statements (II) and (III),

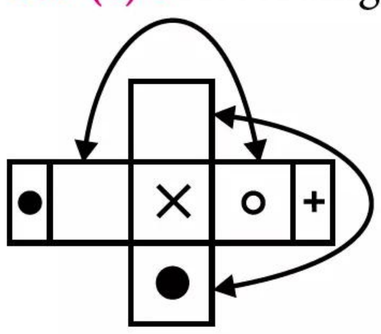
The rank of Animesh is 9^{th}.

11. (d) : From statement (I), E > B > A
From statement (II), □ > □ > □ > C > □ > □
From statement (III), □ > □ > □ > □ > D > F
From all the three statements
E > B > A > C > D > F ∴ E is the tallest

12. (a) : We know that opposites can not appear adjacent to each other. So, by given information, we have blue is opposite to green colour.



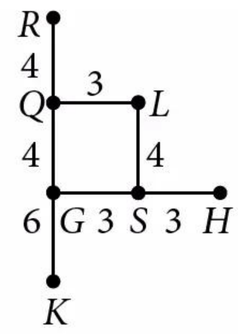
13. (c) : According to the unfolded dice,



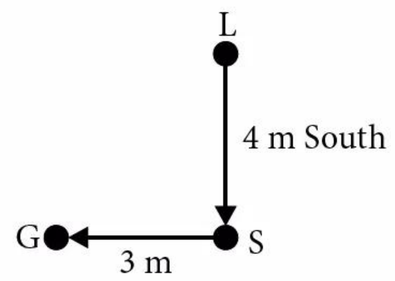
So, □ is opposite to ●
□ is opposite to ○
and ✕ is opposite to ●+

Hence, option (c) is the correct answer.

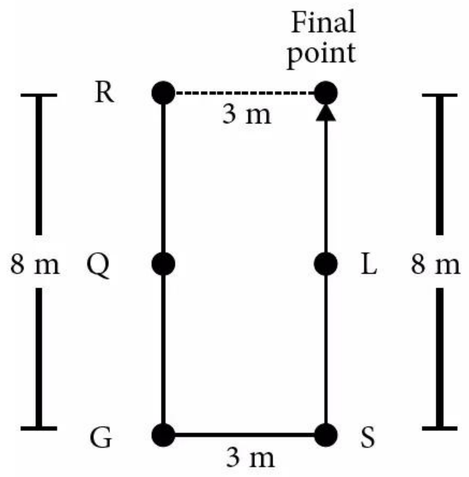
(14-15) : According to given information, we have



14. (b) : Clearly, the person starts from L goes 4 m southward and reaches point S from where he takes right turn and walks 3 m to reach G.



15. (d) :



Clearly, man will cross L and at the final point, he will be at a distance of 3 m from R.

(16-20) :

Person	Gender	Car	Colour
A	Male	*Y*	Grey
B	Female	*Z*	Blue
C	Female	*X*	Red
D	Male	*Z*	Yellow
E	Male	*X*	White
F	Male	*X*	Purple
G	Male	*Z*	Black
H	Female	*Y*	Green

16. (b) : A and H are travelling in the Car *Y*.

17. (d) : *C* is the female member in the Car *X*. She likes Red colour.

18. (a) : *D* likes Yellow colour.

19. (c) : *E* and *F* are travelling with *C*.

20. (d) : The combination *E* - *Y* - Grey is not correct. As *E* travels in Car *X* and he likes white colour.

Class XI

Monthly test drive

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Series-9 Introduction to Three Dimensional Geometry, Limits and Derivatives

Total Marks : 80 **Time taken :** 60 Min.

Only One Option Correct Type

1. Find the ratio in which the line joining the points $A(2, 1, 5)$ and $B(3, 4, 3)$ is divided by the plane $2x + 2y - 2z = 1$.

(a) 3 : 7 (b) 5 : 7 (c) 2 : 7 (d) 1 : 6

2. Given the points P, Q, R, S are (4, 7, 8), (–1, –2, 1), (2, 3, 4) and (1, 2, 5) respectively. If PQ and RS intersect, find the point of intersection.

(a) $\left(\frac{1}{2}, \frac{5}{2}, \frac{9}{2}\right)$ (b) $\left(\frac{9}{2}, \frac{1}{2}, \frac{5}{2}\right)$

(c) $\left(\frac{3}{2}, \frac{5}{2}, \frac{9}{2}\right)$ (d) None of these

3. $\lim\limits_{x \to -7} \dfrac{[x]^2 + 15[x] + 56}{\sin(x+7)\sin(x+8)} =$

(where $[\cdot]$ denotes the greatest integer function)

(a) 0 (b) 1

(c) –1 (d) does not exist

4. If $\lim\limits_{x \to 0} \dfrac{\cos 4x + a\cos 2x + b}{x^4}$ is finite, then $(a, b) =$

(a) (5, –4) (b) (–5, –4) (c) (–4, 3) (d) (4, 5)

5. The value of $\lim\limits_{n \to \infty} \left(\dfrac{\sqrt{n^2+n}-1}{n}\right)^{2\sqrt{n^2+n}-1}$ is

(a) e (b) $1/e$ (c) e^2 (d) e^{-2}

6. If $f(x) = \lambda x^2 + \mu x + 12$, $f'(4) = 15$ and $f'(2) = 11$, then values of λ and μ respectively are

(a) 2, 5 (b) 3, 7 (c) 7, 1 (d) 1, 7

One or More Than One Option(s) Correct Type

7. If the origin is the centroid of the triangle with vertices $P(p, 2, 6)$, $Q(-4, q, -10)$ and $R(8, 14, r)$, then

(a) $p = -4$ (b) $q = -16$

(c) $r = 4$ (d) $p + q + r = 8$

8. Which of the following statements is INCORRECT?

(a) The points $A(2, 3, 4)$, $B(-1, 2, -3)$ and $C(-4, 1, -10)$ are collinear.

(b) (2, 0, 0), (2, 2, 0), (0, 2, 0), (0, 2, 2), (0, 0, 2), (2, 0, 2), (0, 0, 0), (2, 2, 2) are the coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincides with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin.

(c) The triangle ABC with vertices $A(0, 4, 1)$, $B(2, 3, -1)$ and $C(4, 5, 0)$ is not right angled.

(d) $z = c$ represents the plane parallel to xy-plane.

9. Let $g(x) = \begin{cases} x^2, & \text{for } x < 1 \\ x, & \text{for } 1 \le x < 4 \\ 4 - x, & \text{for } x \ge 4 \end{cases}$. Then,

(a) $\lim\limits_{x \to 1^-} g(x) = 1$ (b) $\lim\limits_{x \to 1^+} g(x) = 1$

(c) $\lim\limits_{x \to 4^-} g(x) = 4$ (d) $\lim\limits_{x \to 4^+} g(x) = 4$

10. If $\lim\limits_{x \to 0} (\cos x + a \sin bx)^{1/x} = e^2$ then the values of a and b are

(a) $a = 1, b = 2$ (b) $a = 2, b = 1/2$

(c) $a = 2\sqrt{2}, b = 1/\sqrt{2}$ (d) $a = 4, b = 2$

11. Let $f(x) = \lim\limits_{n \to \infty} \dfrac{\tan^{-1}(\tan x)}{1 + (\log_e x)^n}, x \ne (2n+1)\dfrac{\pi}{2}$, then

(a) $\forall 1 < x < \dfrac{\pi}{2}$, $f(x)$ is an identity function.

(b) $\forall\ \pi/2 < x < \pi$, the graph of $f(x)$ is a straight line having y intercept of $-\pi$.
(c) $\forall\ \pi/2 < x < e$, the graph of $f(x)$ is a straight line having y intercept of $-\pi$.
(d) $\forall\ x > e$, $f(x)$ is a constant function.

12. Given a real-valued function f such that

$$f(x)=\begin{cases}\dfrac{\tan^2\{x\}}{(x^2-[x]^2)}, & \text{for } x>0\\ 1, & \text{for } x=0\\ \sqrt{\{x\}\cot\{x\}}, & \text{for } x<0\end{cases}$$

where $[x]$ is the integral part and $\{x\}$ is the fractional part.

(a) $\lim\limits_{x\to 0^+} f(x)=1$ (b) $\lim\limits_{x\to 0^-} f(x)=\cot 1$

(c) $\cot^{-1}\left(\lim\limits_{x\to 0^-} f(x)\right)^2=1$

(d) $\tan^{-1}\left(\lim\limits_{x\to 0^+} f(x)\right)=\dfrac{\pi}{4}$

13. Assume that $\lim\limits_{\theta\to -1} f(\theta)$ exists and $\dfrac{\theta^2+\theta-2}{\theta+3}\le\dfrac{f(\theta)}{\theta^2}\le\dfrac{\theta^2+2\theta-1}{\theta+3}$ holds for certain interval containing the point $\theta=-1$, then $\lim\limits_{\theta\to -1} f(\theta)$ is

(a) equal to $f(-1)$ (b) equal to 1
(c) non-existent (d) equal to -1

Comprehension Type

Paragraph for Q. No. 14 and 15

$\lim\limits_{x\to 0}\dfrac{1-\cos\left(f(x)\right)}{\left(f(x)\right)^2}=\dfrac{1}{2}$ if $\lim\limits_{x\to 0} f(x)=0$ and α, β, γ are the real roots of $ax^3+bx^2+cx+d=0$,

14. $\lim\limits_{x\to\alpha}\dfrac{1-\cos\left(ax^3+bx^2+cx+d\right)}{(x-\alpha)^2}=$

(a) $a^2(\alpha-\beta)^2(\alpha-\gamma)^2$ (b) $a(\alpha-\beta)(\alpha-\gamma)$
(c) $a(\alpha\beta+\beta\gamma+\gamma\alpha)$ (d) $a^2/2(\alpha-\beta)^2(\alpha-\gamma)^2$

15. $\lim\limits_{x\to 0}\dfrac{1-\cos 1\cdot\cos(\cos x)-\sin 1\cdot\sin(\cos x)}{x^4}=$

(a) 1/8 (b) 1/16 (c) 1/6 (d) 1

Matrix Match Type

16. Match the following.

	Column - I		Column - II
(P)	If the centroid of the triangle is origin and two of its vertices are (3, –5, 7) and (–1, 7, –6), then the third vertex is	(1)	the vertices of a parallelogram
(Q)	If the mid-points of the sides of triangle are (1, 2, –3), (3, 0, 1) and (–1, 1, –4), then the centroid is	(2)	(–2, –2, –1)
(R)	The points $A(3, -1, -1)$, $B(5, -4, 0)$, $C(2, 3, -2)$ and $D(0, 6, -3)$ are	(3)	(1, 1, –2)
(S)	Points $A(2, 3, 4)$, $B(-1, 2, -3)$ and $C(-4, 1, -10)$ are	(4)	collinear

(a) (P) → (2); (Q) → (3); (R) → (1); (S) → (4)
(b) (P) → (3); (Q) → (2); (R) → (1); (S) → (4)
(c) (P) → (2); (Q) → (4); (R) → (3); (S) → (1)
(d) (P) → (2); (Q) → (1); (R) → (3); (S) → (4)

Numerical Answer Type

17. If the extremities of the diagonal of a square are (1, –2, 3) and (2, –3, 5), then the length of the side is $\sqrt{a}$. Then the value of a is _____.

18. Let $f(x)$ be a polynomial satisfying $\lim\limits_{x\to\infty}\dfrac{x^2 f(x)}{2x^5+3}=6$, $f(1)=3$, $f(3)=7$ and $f(7)=15$. Then the value of $\left[\dfrac{|f(0)|}{30}\right]$, where $[\cdot]$ represents the greatest integer function, is _____.

19. Let $f(x)=\begin{cases}\sin x, & x\neq n\pi\\ 2, & x=n\pi\end{cases}$, where $n\in Z$ and

$g(x)=\begin{cases}x^2+1, & x\neq 2\\ 3, & x=2\end{cases}$, then $\lim\limits_{x\to 0} g[f(x)]$ is _____.

20. If $y=\dfrac{\sin x+\cos x}{\sin x-\cos x}$, then $\dfrac{dy}{dx}$ at $x=0$ is $(-n)$, then n is _____.

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted .....
No. of questions correct .....
Marks scored in percentage .....

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CBSE

Warm-up!

CLASS-XII

Practice Paper for CBSE Exams as per the latest pattern and syllabus by CBSE for the academic session 2024-25.

PRACTICE PAPER

General Instructions : Read the following instructions very carefully and strictly follow them :

(i) This question paper contains **38** questions. **All** questions are **compulsory**.

(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.

(iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and Questions no. **19 and 20** are Assertion-Reason based questions of **1** mark each.

(iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.

(v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.

(vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions, carrying **5** marks each.

(vii) In **Section E**, Questions no. **36** to **38** are case study based questions, carrying **4** marks each.

(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B**, **3** questions in Section **C**, **2** questions in Section **D** and **2** questions in Section E.

(ix) Use of calculators is **not** allowed.

Time Allowed : 3 hours **Maximum Marks :** 80

SECTION - A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The value of $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ is

(a) 0 (b) 1 (c) 2 (d) 3

2. If $f(x)=\dfrac{x^3+3x+5}{x^3-3x+2}$, then f is

(a) continuous except at $x = 1, x = -2$

(b) continuous except at $x = 1$

(c) continuous except at $x = -2$

(d) discontinuous everywhere

3. $\int (e^{x\log a}+e^{a\log x}+e^{a\log a})\,dx$ is equal to

(a) $\dfrac{a^x}{\log a}+\dfrac{x^{a+1}}{a+1}+a^a x+c$

(b) $\dfrac{a^x}{\log a}+\dfrac{x^{a+1}}{a-1}+ax^a+c$

(c) $\dfrac{a^x}{\log a}+\dfrac{x^a}{a+1}+ax^a+c$

(d) $\dfrac{a^x}{\log x}+\dfrac{x^{a+1}}{a+1}+a^a x+c$

4. If $A=\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then $\alpha =$

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 5

5. If $\sqrt{\tan\sqrt{x}}$, then find $\dfrac{dy}{dx}$.

(a) $\dfrac{-\sin^2\sqrt{x}}{4\sqrt{x}\sqrt{\tan\sqrt{x}}}$ (b) $\dfrac{\sec^2\sqrt{x}}{4\sqrt{x}\sqrt{\tan\sqrt{x}}}$

(c) $\dfrac{-\sec^2\sqrt{x}}{\sqrt{x}\sqrt{\tan\sqrt{x}}}$ (d) $\dfrac{\sec^2\sqrt{x}}{\sqrt{x}\sqrt{\tan\sqrt{x}}}$

6. A vector of magnitude 7 units, parallel to the resultant of the vectors $\vec{a}=2\hat{i}-3\hat{j}-2\hat{k}$ and $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$, is

(a) $\frac{7}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ (b) $(\hat{i}-\hat{j}-\hat{k})$

(c) $\frac{7}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$ (d) $\frac{7}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

7. Find the order of the differential equation $(xy^2 + x)dx + (y - x^2y)dy = 0$.

(a) 3 (b) not defined

(c) 1 (d) 2

8. Direction ratios of the line segment joining the two points $P(3, -2, -5)$ and $Q(3, -2, 6)$ is

(a) 0, 1, –11 (b) 0, 0, 11

(c) –11, 0, 11 (d) 6, – 4, 1

9. If $\begin{bmatrix} x+y & 2 \\ 1 & x-y \end{bmatrix}=\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$, then values of x and y are respectively

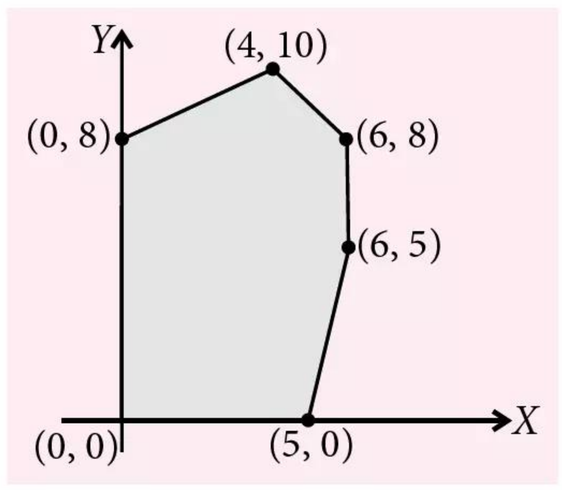
(a) $x = -2, y = 5$ (b) $x = 5, y = -2$

(c) $x = 5, y = 5$ (d) $x = 5, y = 2$

10. Given $P(A) = 0.4$, $P(B) = 0.7$ and $P(B|A) = 0.6$. Find $P(A \cup B)$.

(a) 0.4 (b) 0.6 (c) 0.15 (d) 0.86

11. The feasible solution for an L.P.P. is shown in following figure. Let $Z = 3x - 4y$ be the objective function. Minimum of Z occurs at



(a) (0, 0) (b) (0, 8)

(c) (5, 0) (d) (4, 10)

12. If A is a 3×3 matrix such that $|5 \cdot \text{adj } A| = 5$, then $|A|$ is equal to

(a) ± 1 (b) $\pm 1/5$ (c) $\pm 1/25$ (d) ± 5

13. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively and $\vec{a} \cdot \vec{b} = 2\sqrt{3}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{2}$

14. Solution of the differential equation $y - x\frac{dy}{dx} = 0$ is

(a) $xy = c$ (b) $y = xc$ (c) $xy = 0$ (d) $y = x$

15. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then :

(a) z is maximum at (2, 72), minimum at (15, 20)

(b) z is maximum at (15, 20), minimum at (40, 15)

(c) z is maximum at (40, 15), minimum at (15, 20)

(d) z is maximum at (40, 15), minimum at (2, 72)

16. If $f(x) = \begin{cases} 2x+1, \text{ when } 1 \le x \le 2 \\ x^2+1, \text{ when } 2 < x \le 3 \end{cases}$, then $\int_1^3 f(x)dx$, equals

(a) $\frac{34}{3}$ (b) $\frac{23}{3}$ (c) $\frac{33}{4}$ (d) $\frac{43}{3}$

17. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=2, |\vec{b}|=3, |\vec{c}|=5$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is

(a) 0 (b) 1 (c) –19 (d) 38

18. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32} \cdot A_{33}$.

(a) 110 (b) –90 (c) –110 (d) 90

ASSERTION – REASON BASED QUESTIONS

Questions number 19 and 20 are Assertion Reason based questions carry 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the options (a), (b), (c) and (d) options as given below.

(a) Both (A) and (R) are true and (R) is the correct explanation of the (A).

(b) Both (A) and (R) are true, but (R) is not the correct explanation of the (A).

(c) (A) is true, but (R) is false.

(d) (A) is false, but (R) is true

19. **Assertion (A) :** If $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$ are two sets, then function $f : A \to B$ defined by $f(x) = x + 2\ \forall x \in A$, then the function f is one-one only.

Reason (R) : A function $f: A \to B$ is said to be one-one if the images of distinct elements of A under f are distinct in B.

20. **Assertion (A) :** The angle between two vector $\vec{a}$ and $\vec{b}$ with magnitude 1 and 2 respectively such that $\vec{a} \cdot \vec{b} = 1$ is $\frac{\pi}{3}$.

Reason (R) : Cosine of angle between two vectors is given by $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$.

SECTION - B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21. Find the domain of $f(x) = \cos^{-1}(1 - x^2)$. Also, find its range.

22. (a) Solve the differential equation $dy = \cos x\,(2 - y \operatorname{cosec} x)\,dx$ given that $y = 2$ when $x = \frac{\pi}{2}$.

OR

22. (b) Find the general solution of the following differential equation : $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

23. (a) Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

OR

23. (b) Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

24. Show that the function defined by $f(x) = \cos(x^2)$ is a continuous function.

25. Find the point of intersection of the lines

$$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1} \text{ and } \frac{x+3}{-36} = \frac{y-3}{2} = \frac{z-6}{4}.$$

SECTION - C

This section comprises of 6 short answer (SA) type questions of 3 marks each.

26. If $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)dx$, then

Find $\int_{-a}^{a} \{f(x) - f(-x)\}\,dx$.

27. (a) One-thirds of the students of a class are boys and the rest are girls. It is known that the probability of a girl getting a first class marks in Board's Exam is 0.6 and a boy getting first class marks is 0.35. Find the probability that a student chosen at random will get first class marks in exam.

OR

27. (b) A problem is given to three students whose probabilities of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{6}$ respectively. If the events of solving the problem are independent, then find the probability

(i) that at least one of them solves it.

(ii) that at least two solve it.

(iii) exactly one of them solve it.

28. (a) If $\int (1+x^4)^{-\frac{1}{4}}\,dx$ is equal to

$$A \ln \left| \frac{(1+x^4)^{\frac{1}{4}} + x}{(1+x^4)^{\frac{1}{4}} - x} \right| + B \tan^{-1}\left(\frac{(1+x^4)^{\frac{1}{4}}}{x} \right) + c,$$

then find the value of $A + B$.

OR

28. (b) Find the value of $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx$.

29. (a) Solve $y\,dx - (x + 2y^2)dy = 0$.

OR

29. (b) Find the solution of the differential equation

$$\frac{x\frac{dy}{dx} - y}{\sqrt{x^2 - y^2}} = 10x^2.$$

30. Solve graphically the following linear programming problem :

Maximise $z = 6x + 3y$,

Subject to the constraints

$4x + y \geq 80$,

$3x + 2y \leq 150$,

$x + 5y \geq 115$,

$x \geq 0, y \geq 0$.

31. Find : $\int \frac{x^2}{(x^2+4)(x^2+9)}\,dx$

SECTION - D

This section comprises of 4 long answer (LA) type questions of 5 marks each.

32. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

33. (a) Show that a function $f : R \to R$ defined by $f(x) = \dfrac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

OR

33. (b) Show that the relation S in set $\mathbb{R}$ of real numbers defined by $S = \{(a, b) : a \leq b^3, a \in \mathbb{R}, b \in \mathbb{R}\}$ is neither reflexive, nor symmetric, nor transitive.

34. Find the product BA of matrices $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it in solving the equations : $x + y + 2z = 1$; $3x + 2y + z = 7$; $2x + y + 3z = 2$.

35. (a) Find the image of the point $(2, -1, 5)$ in the line $\dfrac{x-11}{10} = \dfrac{y+2}{-4} = \dfrac{z+8}{-11}$. Also find the equation of the line joining the given point and its image. Find the length of that line segment also.

OR

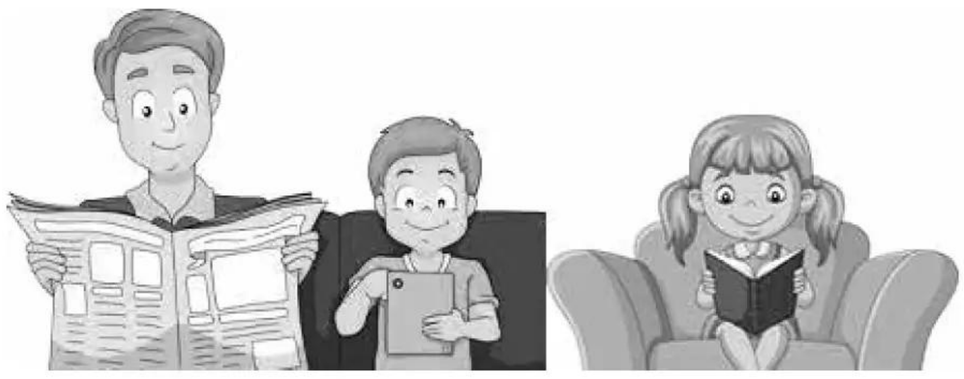
35. (b) Find the direction cosines of the sides of the triangle whose vertices are $P(0, 7, 10)$, $Q(-1, 6, 6)$ and $R(-4, 9, 6)$.

SECTION - E

This section comprises of 3 Case - study / passage - based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case-study question has two subparts of 2 marks each.

Case Study 1

36. On a holiday, a father gave a puzzle from a newspaper to his son Ravi and his daughter Priya. The probability of solving this specific puzzle independently by Ravi and Priya are $\dfrac{1}{4}$ and $\dfrac{1}{5}$ respectively.



On the basis of above information, answer the following question.

(i) Find the probability that both Ravi and Priya solved the puzzle.

(ii) Find the probability that puzzle is solved by Ravi but not by Priya.

(iii) (a) Find the probability that puzzle is solved.

OR

(iii) (b) Find the probability that exactly one of them solved the puzzle.

Case Study 2

37. In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F, where the room temperature is 50°F. Also, it is given the body temperature at the time of death was normal, *i.e.*, 98.6°F.

Let T be the temperature of the body at any time t and initial time is taken to be 8 p.m.

On the basis of above information, answer the following question.

(i) If $\dfrac{dT}{dt} = kf(T)$, where k is constant, then what is the value of $f(T)$?

(ii) What is the body temperature when $t = 0$ and $t = 2$?

(iii) (a) Find the value of $T - 50$ in terms of k.

OR

(iii) (b) Find the value of T at any time t.

Case Study 3

38. A real estate company is going to build a new residential complex. The land they have purchased can hold at most 4500 apartments. Also, if they make x apartments, then the monthly maintenance cost for the whole complex would be as follows : Fixed cost = ₹ 50,00,000, Variable cost = ₹$(160x - 0.04x^2)$.

On the basis of above information, answer the following question.

(i) If $C(x)$ denote the maintenance cost function, then find the maximum value of $C(x)$ occur at x.

(ii) Find the number of apartments that the complex should have in order to minimize the maintenance cost.

SOLUTIONS

1. (b) : We have, $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1$

2. (a) : f is not defined when

$x^3 - 3x + 2 = 0 \Rightarrow (x-1)^2(x+2) = 0 \Rightarrow x = 1, -2$

$\Rightarrow$ f is continuous everywhere except at $x = 1, -2$

3. (a) : Let $I = \int (e^{x\log a} + e^{a\log x} + e^{a\log a})\,dx$

$= \int (e^{\log a^x} + e^{\log x^a} + e^{\log a^a})\,dx$

$= \int (a^x + x^a + a^a)\,dx \qquad [\because e^{\log\lambda} = \lambda]$

$= \int a^x dx + \int x^a dx + \int a^a dx = \dfrac{a^x}{\log a} + \dfrac{x^{a+1}}{a+1} + a^a x + c$

4. (c) : We have

$$A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} = \alpha^2 - 4$$

Now, $|A^3| = |A|^3 = 125 = 53$

$\therefore$ $|A| = 5 \Rightarrow \alpha^2 - 4 = 5 \quad \Rightarrow \quad \alpha^2 = 9 \Rightarrow \alpha = \pm 3$

5. (b) : Let $y = \sqrt{\tan\sqrt{x}}$

$\therefore$ $\dfrac{dy}{dx} = \dfrac{d}{dx}(\sqrt{\tan\sqrt{x}})$

$= \dfrac{1}{2\sqrt{\tan\sqrt{x}}} \cdot \dfrac{d}{dx}(\tan\sqrt{x})$

$= \dfrac{1}{2\sqrt{\tan\sqrt{x}}} \times \sec^2\sqrt{x} \cdot \dfrac{d}{dx}(\sqrt{x})$

$= \dfrac{1}{2\sqrt{\tan\sqrt{x}}} \times \sec^2\sqrt{x} \times \dfrac{1}{2\sqrt{x}} = \dfrac{\sec^2\sqrt{x}}{4\sqrt{x}\sqrt{\tan\sqrt{x}}}$

6. (d) : Let $\vec{r}$ be the vector, where $|\vec{r}| = 7$

Resultant of the vector $\vec{a}$ and $\vec{b}$

$= (2-1)\hat{i} + (-3+2)\hat{j} + (-2+1)\hat{k} = \hat{i} - \hat{j} - \hat{k}$

Vector parallel to resultant vector $= \dfrac{7(\hat{i} - \hat{j} - \hat{k})}{\sqrt{1+1+1}}$

$= \dfrac{7}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

7. (c) : The given differential equation can be written as

$\dfrac{dy}{dx} = \dfrac{-(xy^2 + x)}{y - x^2 y}$

$\therefore$ Order of differential equation is 1.

8. (b) : The direction ratios of the line PQ are

$< 3-3, -2-(-2), 6-(-5) >$ *i.e.* $< 0, 0, 11 >$ or $< 0, 0, -11 >$

9. (b) : By definition of equal matrices, we have

$x + y = 3$ and $x - y = 7$

Solving these two equations, we get

$2x = 10 \Rightarrow x = 5$ and $5 + y = 3 \Rightarrow y = -2$

$\therefore$ Values of x and y are respectively 5, –2.

10. (d) : We have, $P(A) = 0.4$, $P(B) = 0.7$ and $P(B \mid A) = 0.6$

Now, $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$

$\Rightarrow$ $P(A \cap B) = P(B \mid A) \times P(A)$

$\Rightarrow$ $P(A \cap B) = 0.6 \times 0.4 = 0.24$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow$ $P(A \cup B) = 0.4 + 0.7 - 0.24 \Rightarrow P(A \cup B) = 0.86$

11. (b) : We have,

Corner Points	**Value of $Z = 3x - 4y$**	
(0, 0)	$3 \times 0 - 4 \times 0 = 0$	
(5, 0)	$3 \times 5 - 4 \times 0 = 15$	← Maximum

(6, 5)	$3 \times 6 - 4 \times 5 = -2$
(6, 8)	$3 \times 6 - 4 \times 8 = -14$
(4, 10)	$3 \times 4 - 4 \times 10 = -28$
(0, 8)	$3 \times 0 - 4 \times 8 = -32$

← Minimum

Hence, the minimum of Z occurs at (0, 8) and minimum value is –32.

12. (b) : We have, $|5 \cdot \text{adj } A| = 5 \Rightarrow 5^3 \cdot |\text{adj } A| = 5$

$\Rightarrow \quad |A|^{3-1} = \frac{1}{5^2} \Rightarrow |A|^2 = 5^{-2} \Rightarrow |A| = \pm\frac{1}{5}$

13. (b) : Given, $\vec{a} \cdot \vec{b} = 2\sqrt{3}, |\vec{a}| = \sqrt{3}, |\vec{b}| = 4$

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 2\sqrt{3} = \sqrt{3} \cdot 4 \cdot \cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

14. (b) : We have, $y - x\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{y} = \frac{dx}{x}$

Integrate on both sides, we get $\int \frac{dy}{y} = \int \frac{dx}{x}$

$\Rightarrow \quad \log y = \log x + \log c \Rightarrow \log y = \log (xc) \Rightarrow y = xc$

15. (c) : The objective function is given as $z = 18x + 9y$

The corner points are given as (2, 72), (15, 20) and (40, 15)

At (2, 72), $z = 18 \times 2 + 9 \times 72 = 36 + 648 = 684$

At (15, 20), $z = 18 \times 15 + 9 \times 20 = 270 + 180 = 450$

At (40, 15), $z = 18 \times 40 + 9 \times 15 = 720 + 135 = 855$

$\therefore$ z is maximum at (40, 15) and minimum at (15, 20).

16. (a) : We have, $\int_1^3 f(x)dx = \int_1^2 (2x+1)dx + \int_2^3 (x^2+1)dx$

$$= \left[\frac{2x^2}{2} + x\right]_1^2 + \left[\frac{x^3}{3} + x\right]_2^3$$

$$= [(4+2) - (1+1)] + \left[(9+3) - \left(\frac{8}{3} + 2\right)\right] = \frac{34}{3}$$

17. (c) : Given, $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \quad (\vec{a} + \vec{b} + \vec{c})^2 = 0$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow \quad (2)^2 + (3)^2 + (5)^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow \quad \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-1}{2}[4 + 9 + 25] = \frac{-1}{2} \times 38 = -19$

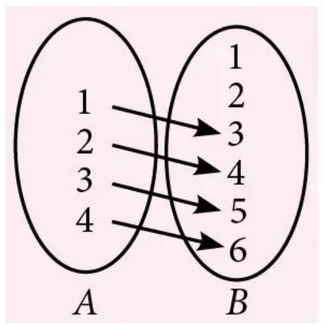
18. (d) : Let $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

Now, $a_{32} = 5$

$A_{33} = \text{cofactor of } a_{33} \text{ in } \Delta = (-1)^{3+3}\begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix} = (0 + 18) = 18$

$\therefore \quad a_{32} \cdot A_{33} = 5 \cdot 18 = 90.$

19. (a) : From the diagram,



Every element of A has distinct image under f in B. Hence, $f(x)$ is one-one.

$\therefore$ Both (A) and (R) true and (R) is the correct explanation of (A).

20. (a) : Given, $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 1$

Now, angle between two vectors is given by

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$\therefore$ Both (A) and (R) are true and (R) is the correct explanation of (A).

21. For domain, $-1 \le 1 - x^2 \le 1, -2 \le -x^2 \le 0$

$\Rightarrow \quad 0 \le x^2 \le 2 \Rightarrow 0 \le |x| \le \sqrt{2} \quad \therefore D_f = \left[-\sqrt{2}, \sqrt{2}\right]$

For range, let $y = \cos^{-1}(1 - x^2)$

$\Rightarrow \quad \cos y = 1 - x^2 \Rightarrow x^2 = 1 - \cos y$

But $0 \le x^2 \le 2 \Rightarrow 0 \le 1 - \cos y \le 2$

$\Rightarrow \quad -1 \le -\cos y \le 1 \Rightarrow -1 \le \cos y \le 1$

$\Rightarrow \quad 0 \le y \le \pi \qquad [\because \cos^{-1} x \text{ is decreasing function}]$

$\therefore \quad R_f = [0, \pi]$

22. (a) We have, $dy = \cos x\,(2 - y\,\text{cosec}\,x)dx$

$\Rightarrow \frac{dy}{dx} = 2\cos x - y\cot x$

$\Rightarrow \frac{dy}{dx} + y\cot x = 2\cos x$, which is a linear differential equation of the type $\frac{dy}{dx} + Py = Q$, where $P = \cot x$ and $Q = 2\cos x$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$

Now, solution is given by

$y \cdot \sin x = \int 2\cos x \cdot \sin x\, dx + C$

$\Rightarrow \quad y \cdot \sin x = \int \sin 2x + C$

$\Rightarrow \quad y \cdot \sin x = -\frac{\cos 2x}{2} + C \qquad \ldots(i)$

At $y = 2$, we have $x = \frac{\pi}{2}$

$\therefore$ From (i), we get

$$2\sin\frac{\pi}{2} = -\frac{\cos\left(2\times\frac{\pi}{2}\right)}{2} + C$$

$$\Rightarrow\ 2\cdot 1 = \frac{1}{2} + C \ \Rightarrow\ C = 2 - \frac{1}{2} = \frac{3}{2}$$

On substituting the value of C in (i), we get

$$y\sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}$$

OR

22. (b) We have, $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$$\Rightarrow\quad dy = \frac{(e^x + x^2)}{e^y}dx \ \Rightarrow\ e^y dy = (e^x + x^2)\,dx$$

Integrating on both sides, we get $\int e^y\,dy = \int (e^x + x^2)\,dx$

$$\Rightarrow\quad e^y = e^x + \frac{x^3}{3} + C,\ \text{which is required solution.}$$

23. (a) We have, $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

$$\therefore\quad \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= (-14+14)\,i - (2-21)\,j + (-2+21)\,k = 19\,j + 19\,k.$$

$$\therefore\quad |\vec{a}\times\vec{b}| = \sqrt{(19)^2 + (19)^2} = 19\sqrt{1+1} = 19\sqrt{2}.$$

OR

23 (b) Let $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$.

$$\Rightarrow\ |\vec{b}| = \sqrt{7^2 + (-1)^2 + 8^2} = \sqrt{49+1+64} = \sqrt{114}$$

and $\vec{a}\cdot\vec{b} = (\hat{i} + 3\hat{j} + 7\hat{k})\cdot(7\hat{i} - \hat{j} + 8\hat{k}) = 7 - 3 + 56 = 60$

$$\therefore\quad \text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{60}{\sqrt{114}}$$

24. Let $f(x) = \cos(x^2)$

Domain of $f = R$.

Let a be any arbitrary real number.

$$\lim_{x\to a^-} f(x) = \lim_{h\to 0}\cos(a-h)^2 = \cos a^2$$

$$\lim_{x\to a^+} f(x) = \lim_{h\to 0}\cos(a+h)^2 = \cos a^2$$

Also, $f(a) = \cos a^2$

Thus, $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x) = f(a)\ \forall\ a \in R.$

$\therefore\quad f(x) = \cos(x^2)$ is continuous $\forall\ \ a \in R.$

25. The given equation of lines are

$$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1} = k\ (\text{say}) \qquad \ldots(\text{i})$$

and $\dfrac{x+3}{-36} = \dfrac{y-3}{2} = \dfrac{z-6}{4}$ $\qquad \ldots(\text{ii})$

$\therefore$ Any point on line (i) is $(3k + 5, 7 - k, k - 2)$.

For some value of k it should lie on line (ii) as lines are intersecting.

$$\therefore\quad \frac{3k+5+3}{-36} = \frac{7-k-3}{2} = \frac{k-2-6}{4}$$

On solving, we get $k = \frac{16}{3}$

$\therefore$ Coordinates of intersection point are $\left(21, \frac{5}{3}, \frac{10}{3}\right)$.

26. We know, $\int_0^{2a} f(x)\,dx = \int_0^a \{f(x) + f(2a-x)\}\,dx$

$$\Rightarrow\quad \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx = 2\int_0^a f(x)\,dx \qquad [\text{Given}]$$

$$\Rightarrow\quad \int_0^a f(2a-x)\,dx = \int_0^a f(x)\,dx \ \Rightarrow\ f(2a-x) = f(x)$$

Now, $I = \int_{-a}^{a} \{f(x) - f(-x)\}\,dx$

For any f, the function $f(x) - f(-x)$ is always an odd function.

$$\therefore\quad \int_{-a}^{a} \{f(x) - f(-x)\}dx = 0$$

27. (a) Let E_1, E_2 and A be the events defined as follows:

E_1 : a boy is chosen,

E_2 : a girl is chosen and

A : the student gets first class marks

Then, $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{2}{3}$

Note that E_1 and E_2 are mutually exclusive and exhaustive events.

$$P(A|E_1) = 0.35 = \frac{35}{100} = \frac{7}{20}$$

$$P(A|E_2) = 0.6 = \frac{6}{10} = \frac{3}{5}$$

By using law of total probability, we get

$$P(A) = P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2)$$

$$= \frac{1}{3}\cdot\frac{7}{20} + \frac{2}{3}\cdot\frac{3}{5} = \frac{7}{60} + \frac{2}{5} = \frac{31}{60}$$

OR

27. (b) Let A, B, C denote the events that the problem is solved by first, second and third student, respectively.

Then, $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{6}$.

(i) Required probability $= P(A \cup B \cup C)$

$$=1 - P(\overline{A \cup B \cup C}) = 1 - P(\overline{A} \cap \overline{B} \cap \overline{C})$$

$$= 1 - P(\overline{A}) \cdot P(\overline{B}) \cdot P(\overline{C}) = 1 - \left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{6}\right)$$

$$= 1 - \frac{2}{3} \times \frac{3}{4} \times \frac{5}{6} = 1 - \frac{5}{12} = \frac{7}{12}$$

(ii) Required probability

$$= P(A \cap B \cap C') + P(A' \cap B \cap C) + P(A \cap B' \cap C) + P(A \cap B \cap C)$$

$$= P(A) \cdot P(B) \cdot P(C') + P(A') \cdot P(B) \cdot P(C) + P(A) \cdot P(B') \cdot P(C) + P(A) \cdot P(B) \cdot P(C)$$

$$= \frac{1}{3} \cdot \frac{1}{4}\left(1-\frac{1}{6}\right) + \left(1-\frac{1}{3}\right) \cdot \frac{1}{4} \cdot \frac{1}{6} + \frac{1}{3} \cdot \left(1-\frac{1}{4}\right) \cdot \frac{1}{6} + \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{6}$$

$$= \frac{5}{72} + \frac{2}{72} + \frac{3}{72} + \frac{1}{72} = \frac{11}{72}.$$

(iii) Required probability

$$= P[(A \cap B' \cap C') \cup (A' \cap B \cap C') \cup (A' \cap B' \cap C)]$$

$$= P(A)\,P(B')\,P(C') + P(A')\,P(B)\,P(C') + P(A')\,P(B')\,P(C)$$

[$\because$ A, B, C are independent events]

$$= \frac{1}{3} \times \frac{3}{4} \times \frac{5}{6} + \frac{2}{3} \times \frac{1}{4} \times \frac{5}{6} + \frac{2}{3} \times \frac{3}{4} \times \frac{1}{6}$$

$$= \frac{15}{72} + \frac{10}{72} + \frac{6}{72} = \frac{31}{72}$$

28. (a) Let $I = \int (1+x^4)^{-\frac{1}{4}}\,dx = \int \frac{dx}{x\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}$

Let $t^4 = 1 + \frac{1}{x^4}$

$$\therefore I = -\int \frac{t^2}{t^4-1}\,dt = -\frac{1}{2}\int\left(\frac{1}{t^2-1} + \frac{1}{t^2+1}\right)dt$$

(Using partial fractions)

$$= -\frac{1}{4}\ln\left|\frac{t-1}{t+1}\right| - \frac{1}{2}\tan^{-1} t + c$$

$$= -\frac{1}{4}\ln\left|\frac{(1+x^4)^{\frac{1}{4}} - x}{(1+x^4)^{\frac{1}{4}} + x}\right| - \frac{1}{2}\tan^{-1}\left(\frac{(1+x^4)^{\frac{1}{4}}}{x}\right) + c$$

$$= \frac{1}{4}\ln\left|\frac{(1+x^4)^{\frac{1}{4}} + x}{(1+x^4)^{\frac{1}{4}} - x}\right| - \frac{1}{2}\tan^{-1}\left(\frac{(1+x^4)^{\frac{1}{4}}}{x}\right) + c$$

$$\Rightarrow \quad A + B = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

OR

28. (b) Let $I = \int_{-\pi}^{\pi} \frac{2x \sin x\,dx}{1+\cos^2 x}$, dropping odd term

$$I = 4\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\,dx \qquad \ldots(i)$$

$$I = 4\int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}\,dx \qquad \ldots(ii)$$

Adding (i) & (ii), we get

$$2I = 4\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx = 8\pi\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}\,dx$$

$$= 8\pi \tan^{-1}(-\cos x)\Big|_0^{\frac{\pi}{2}} = 8\pi\left(0 + \frac{\pi}{4}\right) = 2\pi^2 \Rightarrow I = \pi^2$$

29. (a) The given differential equation is

$y\,dx - (x + 2y^2)\,dy = 0$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x+2y^2} \Rightarrow \frac{dx}{dy} = \frac{x+2y^2}{y}$$

$$\Rightarrow \quad \frac{dx}{dy} + \left(\frac{-1}{y}\right)x = 2y,$$ which is a linear differential equation of the form.

$$\frac{dx}{dy} + Px = Q, \text{ where } P = -\frac{1}{y} \text{ and } Q = 2y$$

$$\therefore \quad \text{I.F.} = e^{\int P\,dy} = e^{\int \frac{-1}{y}dy} = e^{-\log y} = e^{\log y^{-1}} = \frac{1}{y}$$

$\therefore$ The solution is given by $x \cdot \frac{1}{y} = \int 2\,dy + c$

$$\Rightarrow \quad \frac{x}{y} = 2y + c$$

OR

29. (b) We have, $\frac{x\frac{dy}{dx} - y}{\sqrt{x^2 - y^2}} = 10x^2$

$$\Rightarrow \quad \frac{x\,dy - y\,dx}{x^2} = 10x \cdot \sqrt{1 - \frac{y^2}{x^2}}\,dx \qquad \ldots(i)$$

Putting $\frac{y}{x} = v \Rightarrow \frac{x\,dy - y\,dx}{x^2} = dv$

From (i), we get $\frac{dv}{\sqrt{1-v^2}} = 10x\,dx$

Integrating on both sides, we get $\int \frac{dv}{\sqrt{1-v^2}} = 10\int x\,dx$

$\Rightarrow \sin^{-1} v = \frac{10x^2}{2} + C \Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = 5x^2 + C$

30. We have, $z = 6x + 3y$

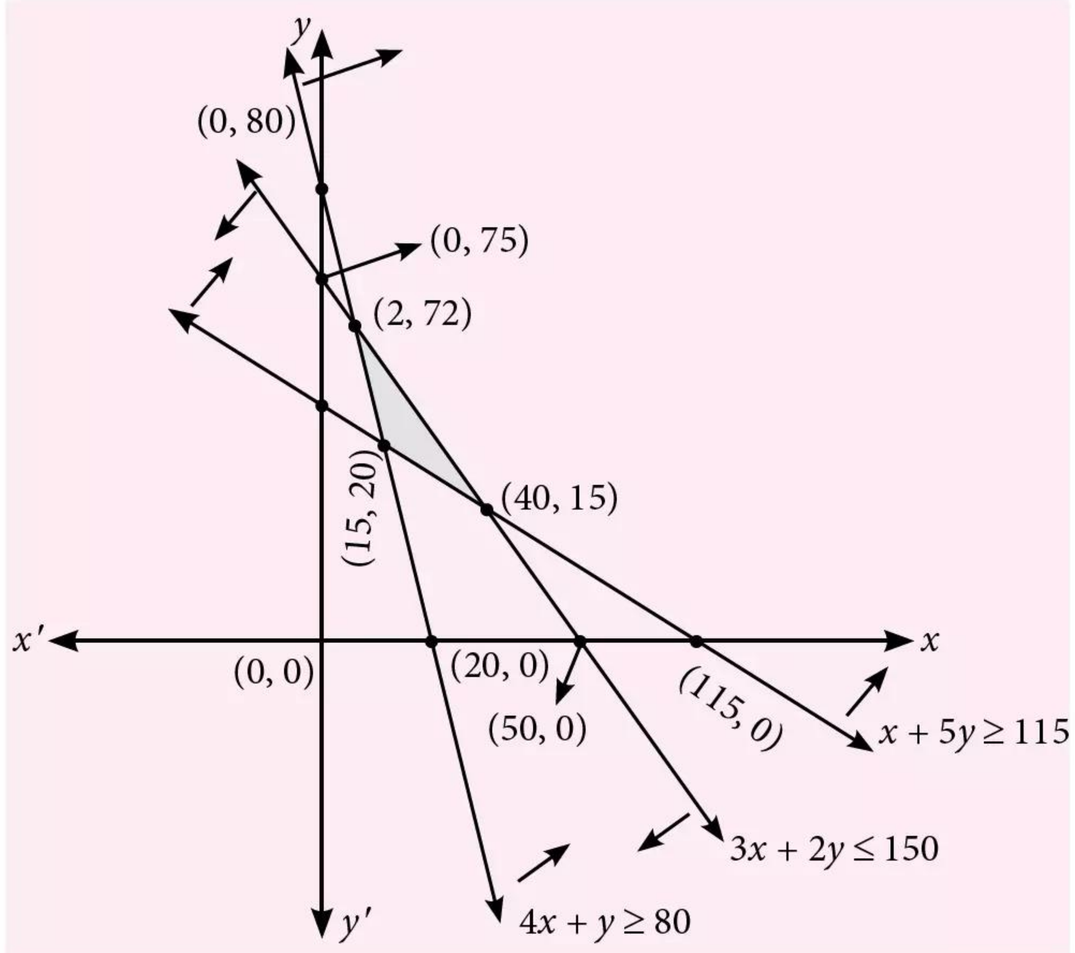
Subject to constraints

$4x + y \geq 80$

$3x + 2y \leq 150$

$x + 5y \geq 115$

$x \geq 0, y \geq 0$



$\therefore$ The corner points are (2, 72), (15, 20) and (40, 15).

Corner points	$z = 6x + 3y$
(2, 72)	$z = 6 \times 2 + 3 \times 72 = 228$
(15, 20)	$z = 6 \times 15 + 3 \times 20 = 150$
(40, 15)	$z = 6 \times 40 + 3 \times 15 = 285$

$\therefore$ The maximum value of $z = 6x + 3y$ is 285 at the corner point (40, 15).

31. Let $I = \int \frac{x^2}{(x^2+4)(x^2+9)}\,dx$

Put $x^2 = y$. Then $\frac{x^2}{(x^2+4)(x^2+9)} = \frac{y}{(y+4)(y+9)}$

Let $\frac{y}{(y+4)(y+9)} = \frac{A}{y+4} + \frac{B}{y+9}$ ...(i)

$\Rightarrow y = A(y+9) + B(y+4)$ ...(ii)

Putting $y = -4$ and $y = -9$ successively in (ii), we get

$A = \frac{-4}{5}$ and $B = \frac{9}{5}$

Substituting the values of A and B in (i), we get

$\frac{y}{(y+4)(y+9)} = \frac{-4/5}{(y+4)} + \frac{9/5}{(y+9)}$

$\Rightarrow \frac{x^2}{(x^2+4)(x^2+9)} = \frac{-4}{5(x^2+4)} + \frac{9}{5(x^2+9)}$

$\therefore I = \int \frac{x^2}{(x^2+4)(x^2+9)}\,dx$

$= \frac{-4}{5}\int \frac{1}{(x^2+4)}\,dx + \frac{9}{5}\int \frac{1}{(x^2+9)}\,dx$

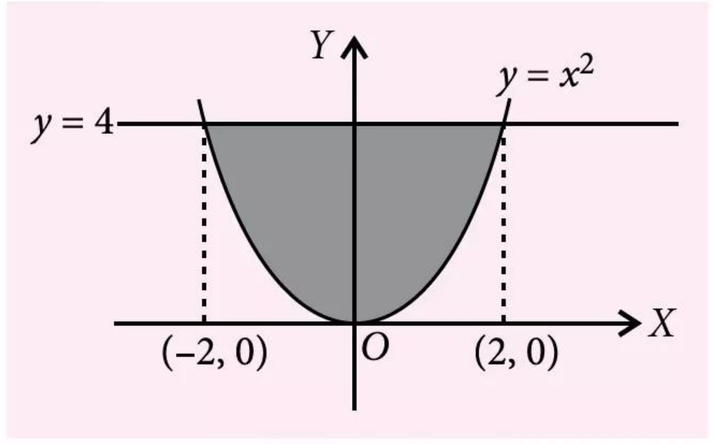
$= \frac{-4}{5} \times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + \frac{9}{5} \times \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$

$= \frac{-2}{5}\tan^{-1}\left(\frac{x}{2}\right) + \frac{3}{5}\tan^{-1}\left(\frac{x}{3}\right) + c$

32. We have, $y = x^2$

and $y = 4$

The graph of $y = x^2$ and $y = 4$ is;



Required area = area of shaded region

$= 2\int_0^2 (4 - x^2)\,dx = 2\left(4x - \frac{x^3}{3}\right)\Big|_0^2 = \frac{32}{3}$ sq. units

33. (a) We have, $f(x) = \frac{2x}{1+x^2}$

For one-one : Let $f(x_1) = f(x_2) \Rightarrow \frac{2x_1}{1+x_1^2} = \frac{2x_2}{1+x_2^2}$

$\Rightarrow x_1 + x_1x_2^2 = x_2 + x_1^2x_2 \Rightarrow (x_1 - x_2) + x_1x_2(x_2 - x_1) = 0$

$\Rightarrow (x_1 - x_2)(1 - x_1x_2) = 0$

$\Rightarrow x_1 = x_2$ or $x_1x_2 = 1$, if $x_1 \neq x_2$

$\Rightarrow f(x)$ is not one-one function.

Foronto: Let $y = \frac{2x}{1+x^2} \Rightarrow y + x^2y - 2x = 0 \Rightarrow yx^2 - 2x + y = 0$

$\Rightarrow x = \frac{2 \pm \sqrt{4 - 4y^2}}{2y} \Rightarrow x = \frac{1 \pm \sqrt{1 - y^2}}{y}$

Now, for $f(x) = 2 = y$, there exists no real value of $x \in R$.

$\therefore$ $f(x)$ is not onto.

We can see that, for $x \in R$, $\sqrt{1-y^2} \geq 0$

$\Rightarrow$ $1 - y^2 \geq 0 \Rightarrow y^2 \leq 1 \Rightarrow -1 \leq y \leq 1$

So, $f(x)$ is onto if $y \in [-1, 1]$

$\therefore$ Required set $A = [-1, 1]$.

OR

33. (b) $S = \{(a, b) : a \leq b^3, a \in R, b \in R\}$

For reflexive : For $(a, a) \in S$, $a \leq a^3$ which is not true.

Let $a = \frac{1}{2}$ then $a^3 = \frac{1}{8}$ but $a \not\leq a^3$

So, S is not reflexive.

For symmetric : For $(a, b) \in S \Rightarrow a \leq b^3$

Let $a = 1$ and $b = 2$

Then, $1 \leq 2^3$ *i.e.*, $a \leq b^3$ but $2 \not\leq 1^3 \Rightarrow (b, a) \notin S$

So, S is not symmetric.

For transitive : for $(a, b), (b, c) \in S$

Let $a = 3 \quad b = 3/2 \quad c = 4/3$

then, $3 \leq \left(\frac{3}{2}\right)^3, \left(\frac{3}{2}\right) \leq \left(\frac{4}{3}\right)^3$ but $3 \not\leq \left(\frac{4}{3}\right)^3$

$\therefore$ S is not transitive.

Hence, S is neither reflexive, nor symmetric nor transitive.

34. We have, $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

So, $BA = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$\Rightarrow$ $BA = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow BA = 4I \Rightarrow B^{-1}(BA) = 4B^{-1}I$

[Pre multiplying by B^{-1}]

$\Rightarrow$ $4B^{-1} = IA \Rightarrow B^{-1} = 1/4A$ ...(i)

Now, given system of equations can be written as

$BX = C$, where

$B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

or $X = B^{-1}C \Rightarrow X = 1/4AC$ [Using (i)]

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$

$\therefore$ $x = 2, y = 1, z = -1$

35. (a) The given line is $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda$ (say) ...(i)

Any point on it is of form,

$M(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$.

Let the given point be $P(2, -1, 5)$

D.r's of PM are $10\lambda + 11 - 2, -4\lambda - 2 + 1, -11\lambda - 8 - 5$.

i.e., $10\lambda + 9, -4\lambda - 1, -11\lambda - 13$.

As line (i) is perpendicular to PM, so

$(10\lambda + 9) \cdot 10 + (-4\lambda - 1) \cdot (-4) + (-11\lambda - 13) \cdot (-11) = 0$

$\Rightarrow$ $(100 + 16 + 121)\lambda + (90 + 4 + 143) = 0$

$\Rightarrow$ $237\lambda + 237 = 0 \Rightarrow \lambda = -1$.

$\therefore$ D.r's of PM are $-1, 3, -2$ *i.e.*, $1, -3, 2$.

$\therefore$ Equation of the line PM is $\frac{x-2}{1} = \frac{y+1}{-3} = \frac{z-5}{2}$ ...(ii)

If $Q(\alpha, \beta, \gamma)$ is the image of P in line (i), then M is the mid point of PQ.

Here, $M \equiv (1, 2, 3)$

$\therefore$ $\frac{\alpha+2}{2} = 1, \frac{\beta-1}{2} = 2, \frac{\gamma+5}{2} = 3 \Rightarrow \alpha = 0, \beta = 5, \gamma = 1$

$\therefore$ $Q(0, 5, 1)$ is the image of P.

and $PQ = \sqrt{(2-0)^2 + (-1-5)^2 + (5-1)^2}$

$= \sqrt{56} = 2\sqrt{14}$ units

OR

35. (b) We have, $|PQ| = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$

$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

$|QR| = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2} = \sqrt{9+9+0}$

$= \sqrt{18} = 3\sqrt{2}$

$|PR| = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2} = \sqrt{16+4+16}$

$= \sqrt{36} = 6$

D.C.'s of PQ are $< \frac{-1-0}{|PQ|}, \frac{6-7}{|PQ|}, \frac{6-10}{|PQ|} >$

i.e., $< \frac{-1}{3\sqrt{2}}, \frac{-1}{3\sqrt{2}}, \frac{-4}{3\sqrt{2}} >$

D.C.'s of QR are $< \frac{-4+1}{|QR|}, \frac{9-6}{|QR|}, \frac{6-6}{|QR|} >$

i.e., $< \frac{-3}{3\sqrt{2}}, \frac{3}{3\sqrt{2}}, \frac{0}{3\sqrt{2}} >$ or $< \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 >$

D.C.'s of PR are $< \frac{-4-0}{|PR|}, \frac{9-7}{|PR|}, \frac{6-10}{|PR|} >$

i.e., $< \frac{-4}{6}, \frac{2}{6}, \frac{-4}{6} >$ or $< \frac{-2}{3}, \frac{1}{3}, \frac{-2}{3} >$

36. Let E_1 be the event that Ravi solved the puzzle and E_2 be the event that Priya solved the puzzle.

Then, $P(E_1) = 1/4$ and $P(E_2) = 1/5$

(i) Since, E_1 and E_2 are independent events.

$\therefore$ P(both solved the puzzle) $= P(E_1 \cap E_2)$

$= P(E_1) \cdot P(E_2) = \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

(ii) P(Puzzle is solved by Ravi but not by Priya)

$= P(\overline{E}_2)P(E_1) = \left(1 - \frac{1}{5}\right) \cdot \frac{1}{4} = \frac{4}{5} \cdot \frac{1}{4} = \frac{1}{5}$

(iii) (a) P(Puzzle is solved) $= P(E_1$ or $E_2)$

$= P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{1}{4} + \frac{1}{5} - \frac{1}{20} = \frac{8}{20} = \frac{2}{5}$

OR

(iii) (b) P(Exactly one of them solved the puzzle)

$= P[(E_1$ and $\overline{E}_2)$ or $(E_2$ and $\overline{E}_1)]$

$= P(E_1 \cap \overline{E}_2) + P(E_2 \cap \overline{E}_1)$

$= P(E_1) \times P(\overline{E}_2) + P(E_2) \times P(\overline{E}_1)$

$= \frac{1}{4} \times \frac{4}{5} + \frac{1}{5} \times \frac{3}{4}$ $[\because P(\overline{E}_1) = 1 - P(E_1)]$

$= \frac{4}{20} + \frac{3}{20} = \frac{7}{20}$

37. (i) Given, T is the temperature of the body at any time t. Then, by Newton's law of cooling, we get

$\frac{dT}{dt} = k(T - 50)$, where k is the constant of proportionality.

$\therefore$ $f(t) = T - 50$

(ii) From given information, we have

At 8 p.m. temperature is 70°F $\therefore$ At $t = 0$, $T = 70$°F

At 10 p.m., temperature is 60°F

$\therefore$ At $t = 2$, $T = 60°$ F

(iii) (a) $\frac{dT}{dt} = k(T - 50) \Rightarrow \frac{dT}{T - 50} = k\ dt$

On integrating both sides, we get

$\log|T - 50| = kt + \log C \Rightarrow T - 50 = Ce^{kt}$

Clearly, for $t = 0$, $T = 70° \Rightarrow C = 20$

Thus, $T - 50 = 20e^{kt}$

OR

(iii) (b) $\frac{dT}{dt} = k(T - 50) \Rightarrow \frac{dT}{T - 50} = k\ dt$

On integrating both sides, we get

$\log|T - 50| = kt + \log C \Rightarrow T - 50 = Ce^{kt}$

Clearly, for $t = 0$, $T = 70° \Rightarrow C = 20$

Thus, $T - 50 = 20e^{kt}$

For $t = 2$, $T = 60° \Rightarrow 10 = 20\ e^{2k}$

$\Rightarrow \quad 2k = \log\left(\frac{1}{2}\right) \Rightarrow k = \frac{1}{2}\log\left(\frac{1}{2}\right)$

Hence, $T = 50 + 20\left(\frac{1}{2}\right)^{\frac{t}{2}}$

38. (i) Let $C(x)$ be the maintenance cost function, then $C(x) = 5000000 + 160\ x - 0.04x^2$

Now, $C'(x) = 160 - 0.08\ x$

For maxima/minima, put $C'(x) = 0$

$\Rightarrow \quad 160 = 0.08x \Rightarrow x = 2000$

Now, $C''(x) = -0.08 < 0$

Hence, at $x = 2000$, $C(x)$ will be maximum and maximum value, $C(x) = 5000000 + 160\ (2000) - 0.04\ (2000)^2$

$= 5000000 + 320000 - 160000 = 5160000$

(ii) Clearly, from the given condition we can see that we only want critical points that are in the interval [0,4500],

Now, we have $C\ (0) = 5000000$

$C\ (2000) = 5160000$

$C\ (4500) = 4910000$

$\therefore$ The complex must have 4500 apartments to minimise the maintenance cost. ◈◈

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(b)	**2.**	(d)	**3.**	(d)	**4.**	(a)	**5.**	(d)
6.	(b)	**7.**	(a,c)	**8.**	(b,c)	**9.**	(a,c)	**10.**	(a,c)
11.	(a,c)	**12.**	(b,d)	**13.**	(b)	**14.**	(c)	**15.**	(d)
16.	(a)	**17.**	(1)	**18.**	(3)	**19.**	(7225)	**20.**	(306)

Class XII

Monthly test drive

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Series-9 Vector Algebra and Three Dimensional Geometry

Total Marks : 80 **Time taken :** 60 Min.

Only One Option Correct Type

1. If the position vectors of the vertices A, B and C of a ΔABC are respectively $4\hat{i}+7\hat{j}+8\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$ and $2\hat{i}+5\hat{j}+7\hat{k}$, then the position vector of the point, where the bisector of $\angle A$ meets BC is

(a) $\frac{1}{2}(4\hat{i}+8\hat{j}+11\hat{k})$

(b) $\frac{1}{3}(6\hat{i}+13\hat{j}+18\hat{k})$

(c) $\frac{1}{4}(8\hat{i}+14\hat{j}+19\hat{k})$

(d) $\frac{1}{3}(6\hat{i}+11\hat{j}+15\hat{k})$

2. If $\vec{a}+2\vec{b}-\vec{c}=\vec{0}$ and $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\lambda(\vec{a}\times\vec{b})$, then the value of λ is equal to

(a) 5 (b) 4 (c) 2 (d) –2

3. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. If $|\vec{c}|=1$ and $(\vec{a}\times\vec{b})\times\vec{c}=0$ then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 =$

(a) 0 (b) 1

(c) $|\vec{a}|^2\,|\vec{b}|^2$ (d) $|\vec{a}\times\vec{b}|^2$

4. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to

(a) 9/2 (b) 0 (c) –1 (d) 2/9

5. A plane which bisects the angle between the two given planes $2x-y+2z-4=0$ and $x+2y+2z-2=0$, passes through the point

(a) (2, 4, 1) (b) (1, 4, –1)

(c) (1, –4, 1) (d) (2, –4, 1)

6. If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane $2x+3y-z+13=0$ at a point P and the plane $3x+y+4z=16$ at a point Q, then PQ is equal to

(a) 14 (b) $2\sqrt{14}$

(c) $2\sqrt{7}$ (d) $\sqrt{14}$

One or More than One Option(s) Correct Type

7. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}, \vec{c}=\hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is

(a) $2\hat{i}+3\hat{j}-3\hat{k}$

(b) $2i+3\hat{j}+3\hat{k}$

(c) $-2\hat{i}-\hat{j}+5\hat{k}$

(d) $2\hat{i}+\hat{j}+5\hat{k}$

8. Let $\hat{u}=u_1\hat{i}+u_2\hat{j}+u_3\hat{k}$ be a unit vector in R^3 and $\hat{w}=\frac{1}{\sqrt{6}}(\hat{i}+\hat{j}+2\hat{k})$. Given that there exists a vector $\vec{v}$ in R^3 such that $|\hat{u}\times\vec{v}|=1$ and $\hat{w}\cdot(\hat{u}\times\vec{v})=1$. Which of the following statement(s) is(are) correct?

(a) There is exactly one choice for such $\vec{v}$.

(b) There are infinitely many choices for such $\vec{v}$.

(c) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$.

(d) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$.

9. Let a and b be positive real numbers. Suppose $\overrightarrow{PQ} = a\hat{i} + b\hat{j}$ and $\overrightarrow{PS} = a\hat{i} - b\hat{j}$ are adjacent sides of a parallelogram $PQRS$. Let $\vec{u}$ and $\vec{v}$ be the projection vectors of $\vec{w} = \hat{i} + \hat{j}$ along $\overrightarrow{PQ}$ and $\overrightarrow{PS}$, respectively. If $|\vec{u}| + |\vec{v}| = |\vec{w}|$ and if the area of the parallelogram $PQRS$ is 8, then which of the following statements is/are TRUE?

(a) $a + b = 4$

(b) $a - b = 2$

(c) The length of the diagonal PR of the parallelogram $PQRS$ is 4

(d) $\vec{w}$ is an angle bisector of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

10. If M is the foot of the perpendicular drawn from the point (2, 3, 4) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Then

(a) $M \equiv \left(\frac{170}{49}, \frac{78}{49}, \frac{10}{49}\right)$

(b) $M \equiv \left(\frac{70}{49}, \frac{78}{49}, \frac{10}{49}\right)$

(c) $PM \equiv \frac{3}{7}\sqrt{101}$ units

(d) $PM \equiv \frac{\sqrt{101}}{7}$ units

11. Consider the line $x = y = z$ and the line $2x + y + z - 1 = 0 = 3x + y + 2z - 2$ then

(a) The shortest distance between the two lines is $\frac{1}{\sqrt{2}}$

(b) The shortest distance between the two lines is $\sqrt{2}$

(c) Plane containing 2^{nd} line parallel to 1^{st} line is $y - z + 1 = 0$

(d) The shortest distance between the two lines is $\frac{\sqrt{3}}{2}$

12. If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the planes containing these two lines are

(a) $y + 2z = -1$ (b) $y + z = -1$

(c) $y + 2z = 1$ (d) $y - z = -1$

Comprehension Type

Paragraph for Q. No. 13 to 15

Given four points $A(2, 1, 0)$, $B(1, 0, 1)$, $C(3, 0, 1)$ and $D(0, 0, 2)$. Point D lies on line L orthogonal to the plane determined by the points A, B and C.

13. The equation of the plane ABC is

(a) $x + y + z - 3 = 0$ (b) $y + z - 1 = 0$

(c) $x + z - 1 = 0$ (d) $2y + z - 1 = 0$

14. The equation of the line L is

(a) $\vec{r} = 2\hat{k} + \lambda(\hat{i} + \hat{k})$

(b) $\vec{r} = 2\hat{k} + \lambda(2\hat{j} + \hat{k})$

(c) $\vec{r} = 2\hat{k} + \lambda(\hat{j} + \hat{k})$

(d) None of these

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	Address	: 406, Taj Apartment, New Delhi - 110029
4.	Editor's Name	: Anil Ahlawat
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Mahabir Singh
Director

15. The perpendicular distance of D from the plane ABC is

(a) $\sqrt{2}$ (b) 1/2

(c) 2 (d) $1/\sqrt{2}$

Matrix Match Type

16. Match the following columns and select the correct option.

Column-I		Column-II	
(P)	If $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=a\vec{\delta}$ and $\vec{\beta}+\vec{\gamma}+\vec{\delta}=b\vec{\alpha}$, $\vec{\alpha}$ and $\vec{\delta}$ are non-collinear, then $\vec{\alpha}+\vec{\beta}+\vec{\gamma}+\vec{\delta}$ equals	(1)	9
(Q)	If the vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are coplanar and $(\sin\alpha)\vec{a}+(2\sin 2\beta)\vec{b}+(3\sin 3\gamma)\vec{c}-\vec{d}=0$ then the least value of $\sin^2\alpha+\sin^2 2\beta+\sin^2 3\gamma$ is	(2)	0
(R)	The negative value of a such that $2\hat{i}-\hat{j}+\hat{k}, \hat{i}+2\hat{j}+(1+a)\hat{k}$ and $3\hat{i}+a\hat{j}+5\hat{k}$ are coplanar is	(3)	$\frac{1}{14}$
(S)	If vectors $\vec{a}=\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+j+2\hat{k}$ are coplanar, then the value of $(\lambda-4)$ is	(4)	−4

(a) (P) → (2); (Q) → (3); (R) → (4); (S) → (1)

(b) (P) → (1); (Q) → (3); (R) → (2); (S) → (4)

(c) (P) → (3); (Q) → (2); (R) → (4); (S) → (1)

(d) (P) → (2); (Q) → (3); (R) → (1); (S) → (4)

Numerical Answer Type

17. Let $\vec{a}, \vec{b}, \vec{c}$ be coplanar unit vectors such that $\vec{b}\cdot\vec{c}=\cos\alpha$, $\vec{c}\cdot\vec{a}=\cos\beta$, $\vec{a}\cdot\vec{b}=\cos\gamma$. Then, the value of $\cos^2\alpha+\cos^2\beta+\cos^2-2\cos\alpha\cos\beta\cos\gamma$ is equal to _____.

18. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}\cdot\vec{b}=0$. For some $x, y \in R$, let $\vec{c}=x\vec{a}+y\vec{b}+(\vec{a}\times\vec{b})$. If $|\vec{c}|=2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is _____.

19. If the position vector of the point of intersection of the line $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+\hat{j}+2\hat{k})$ and the plane $\vec{r}\cdot(2\hat{i}-6\hat{j}+3\hat{k})+5=0$ is $a\hat{i}+b\hat{j}+c\hat{k}$, then $(50a+60b+75c)^2$ is equal to _____.

20. Let S be the set of all values of λ, for which the shortest distance between the lines $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ and $\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ is 13. Then $8\left|\sum_{\lambda\in S}\lambda\right|$ is equal to _____.

1− 7	1− 5	6	1− 4	3	64× 8	2	2− 1
6	4× 1	50× 5	2	15+ 8	4	7 7	3
1− 3	4	1	5	7	2− 6	8	60× 2
4	21× 3	7	7− 8	1	3− 2	5	6
4− 1	4÷ 2	8	2− 6	4	5	168× 3	7
5	224× 7	4	11+ 1	3÷ 2	3− 3	6	8
4÷ 2	8	3	7	6	7× 1	13+ 4	5
8	3÷ 6	2	15× 3	5	7	1	4

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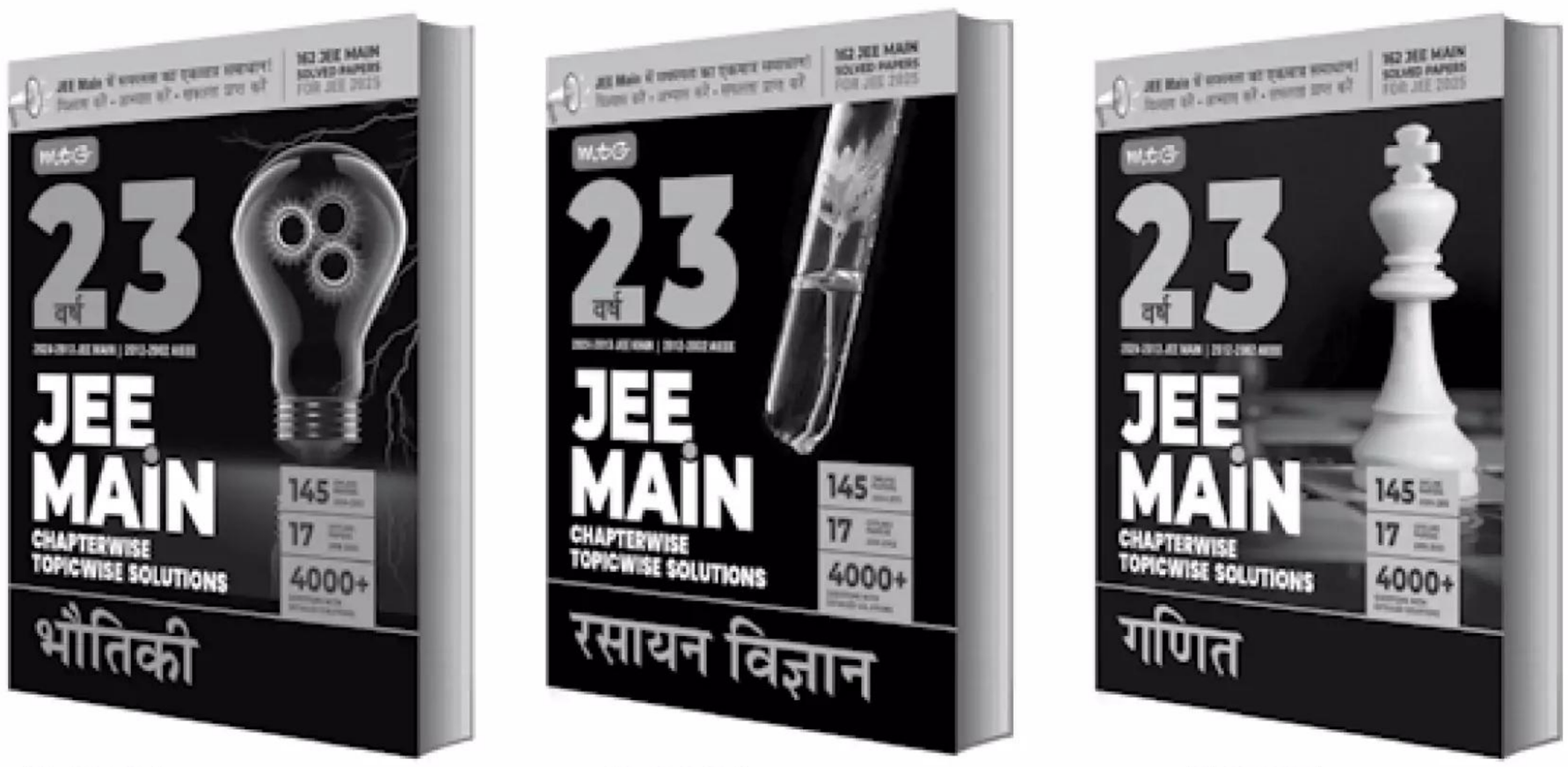

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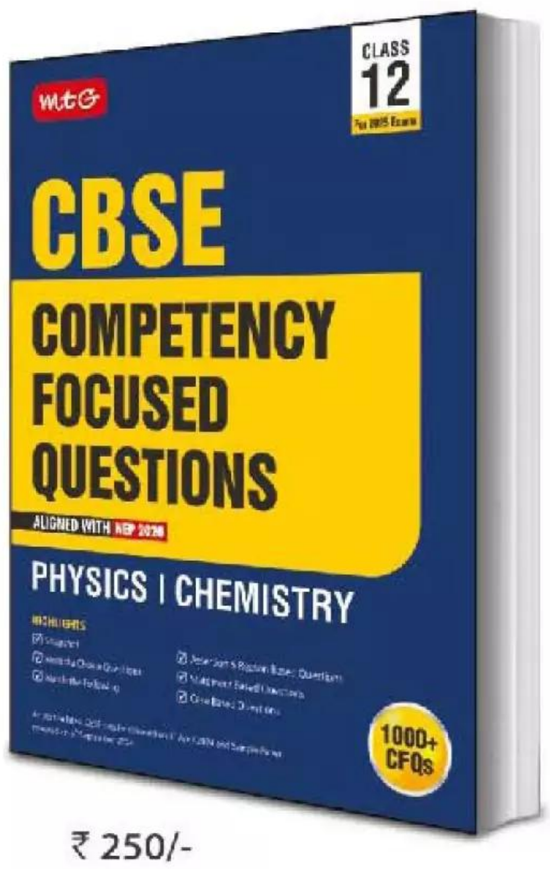

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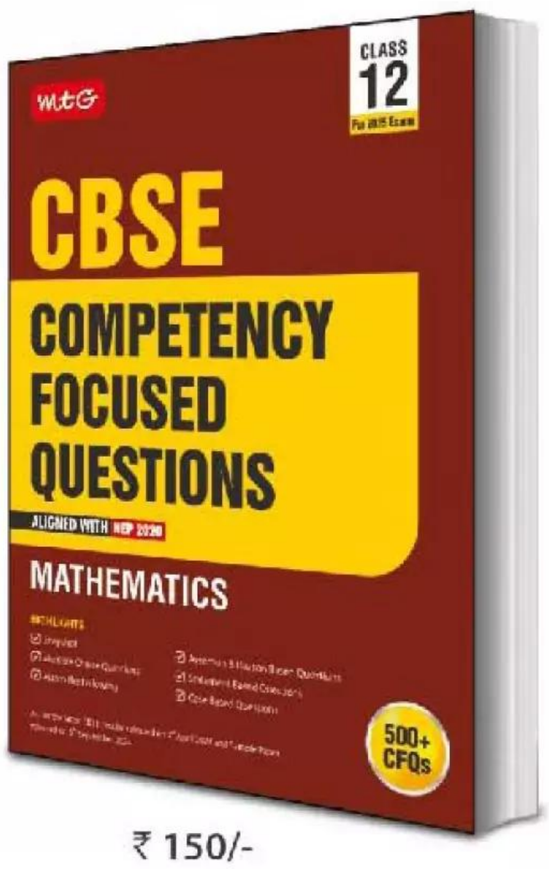

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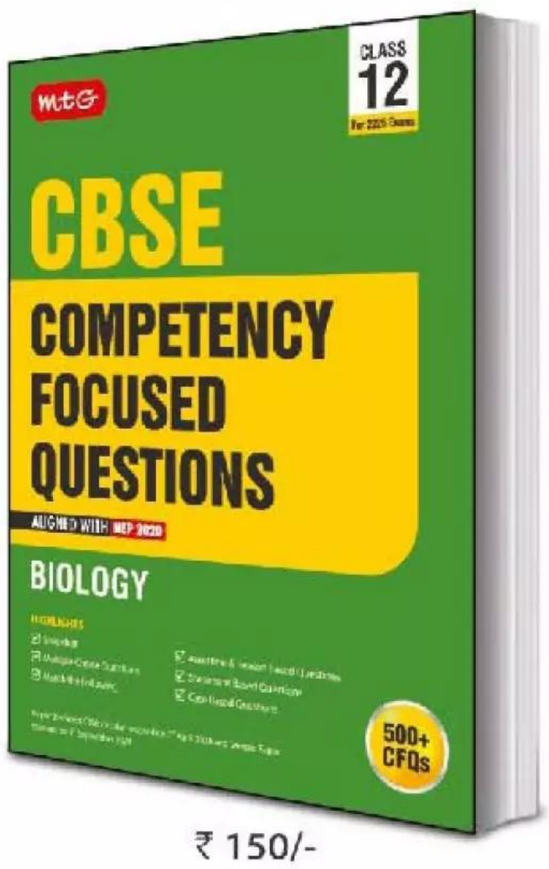

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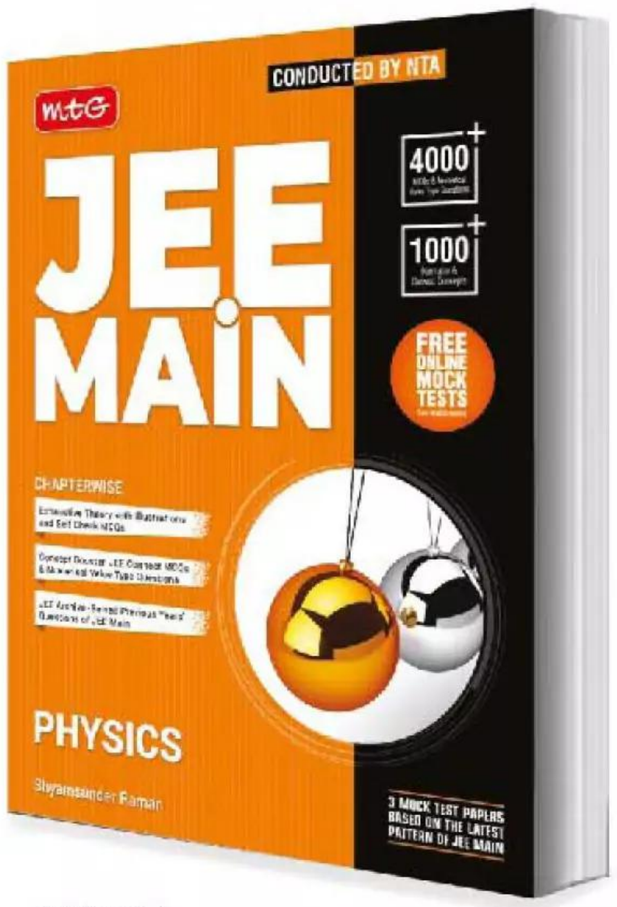

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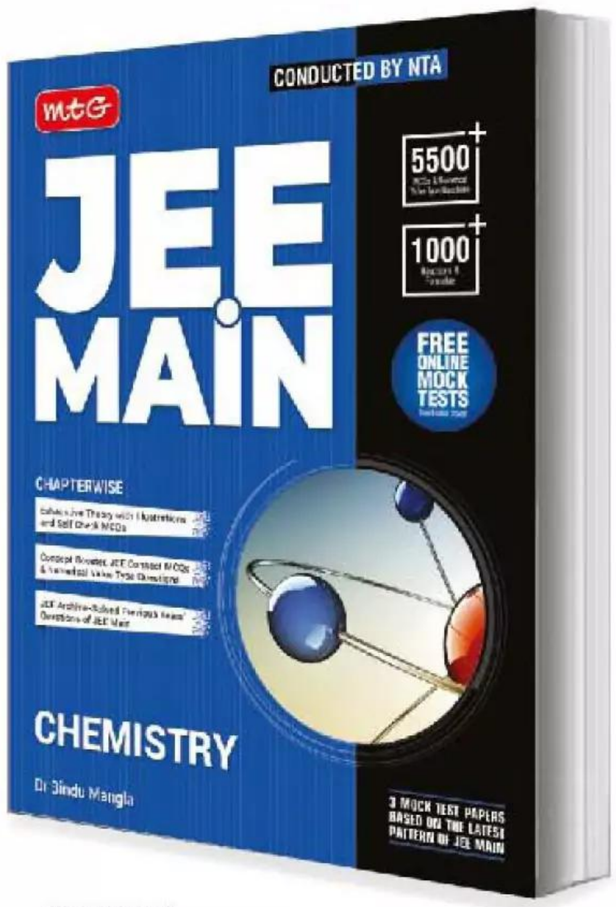

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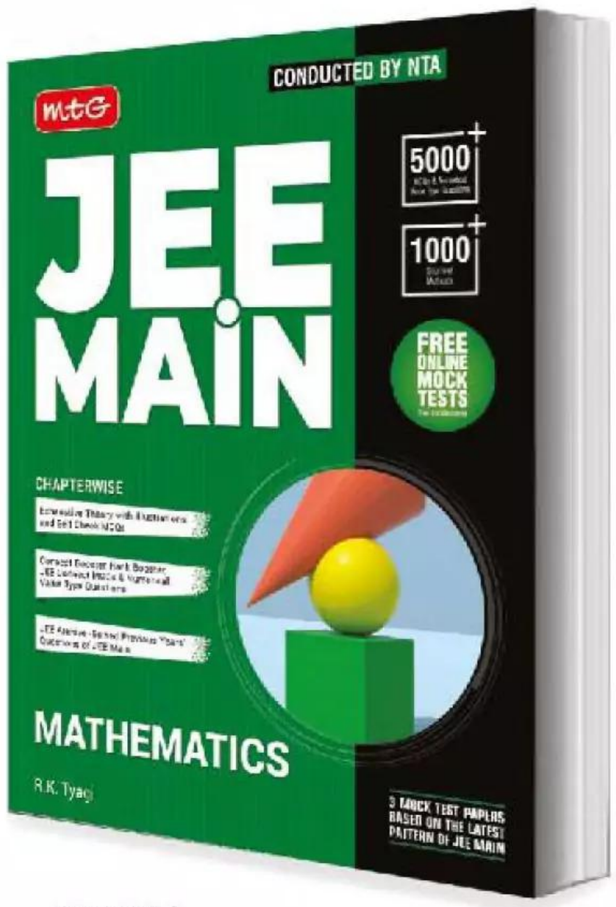

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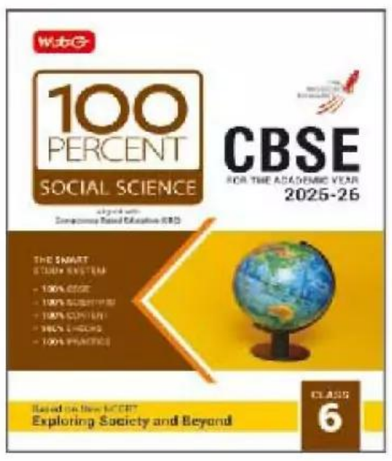

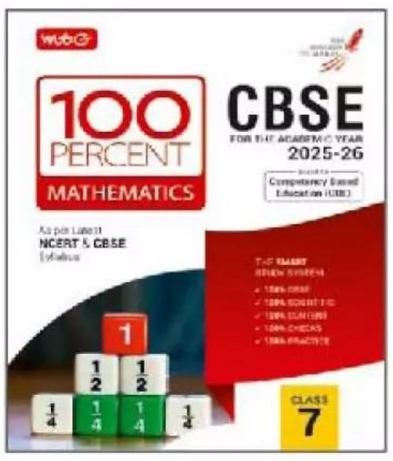

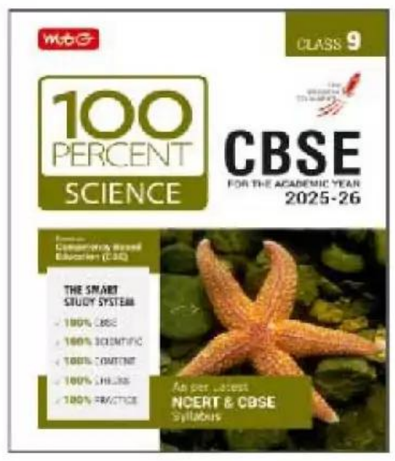

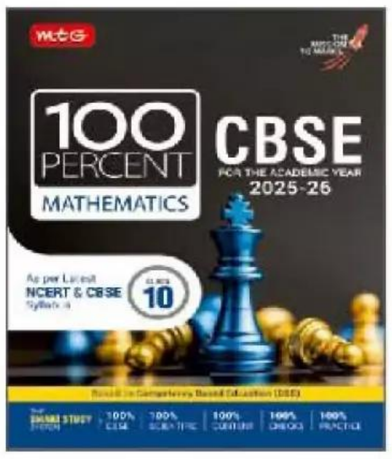

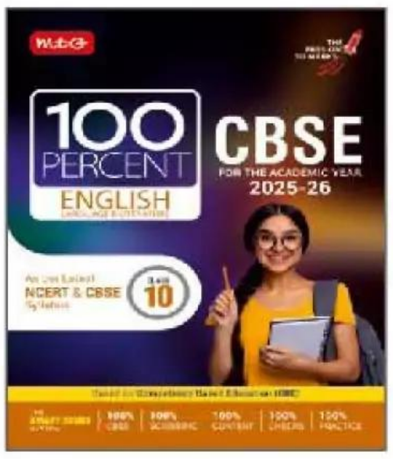

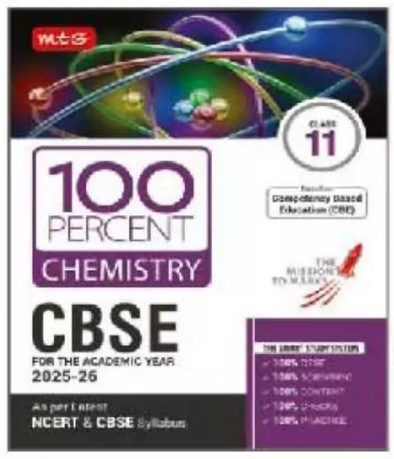

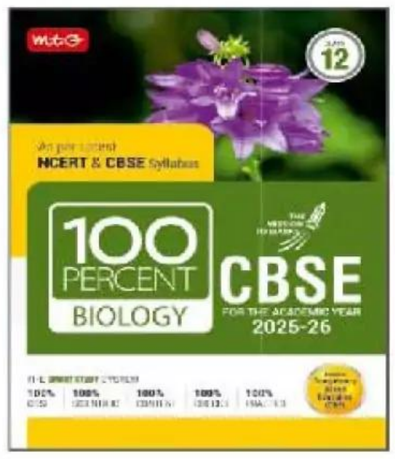

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FIRST BATCH
20.03.2025
THURSDAY

CRASH COURSE
SECOND BATCH
26.03.2025
WEDNESDAY

Note : Registrations Started for Integrated PUC + CET / NEET / IIT-JEE

NEET / CET/ IIT-JEE

#326, 3rd A-Cross, Opp. People's Tree Hospital, Sheshadripuram College Road, Yelahanka New Town, Bengaluru - 560 064, KARNATAKA

7676-91-7777, 95136 57041, 9535656277.

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